A Generally Weighted Moving Average Control Chart for Monitoring the Coefficient of Variation

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Abstract

This paper proposes a generally weighted moving average control chart with adjusted time-varying control limits for monitoring the coefficient of variation of a normally distributed process variable. This control chart is constructed by combining the generally weighted moving average procedure with a resetting model. The implementation of the proposed chart is presented. Some numerical comparison of the proposed chart with several relevant competing control charts is performed. In general, as demonstrated by extensive simulation results, our chart is clearly more sensitive than other competing procedures for each combination of the in-control target value of the coefficient of variation, the sample size and the shift size. Detection examples are given for two industrial manufacturing processes to introduce the proposed control chart.

Keywords: Generally weighted moving average; Adjusted time-varying control limits; Resetting technique; Coefficient of variation; Average run length.

1 Introduction

Control charting techniques are crucial process monitoring tools and have been widely used in practice [1-3]. When we consider variable data, most control charts are used to monitor the process mean or standard deviation. However, not all

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process means or standard deviations are constant even though the process may be smoothly working within an acceptable range of dispersion. In this situation, we usually consider the coefficient of variation (CV), which is defined as the ratio $\gamma$ of the standard deviation $\sigma$ to the mean $\mu$. The CV is a dimensionless measure of the dispersion that is widely applied to measure the relative variation of a variable.

When comparing the variations of several variables, the standard deviation is not a suitable measure of the variation unless the variables are all expressed in identical units of measurement and have the same mean. When the above requirements do not hold, the CV is more meaningful than the standard deviation for comparing the variability among several groups of observations.

Thus, monitoring the CV shifts via a chart has received considerable attention. The first univariate CV control chart was introduced by Kang et al. [4] and was sensitive to large shifts. Hong et al. [5] designed an exponentially weighted moving average (EWMA) CV chart to enhance the sensitivity of the detection. Results based on the average run length (ARL), showed a process improvement in detecting small and moderate shifts. Subsequently, Castagliola et al. [6] proposed a chart based on monitoring the CV squared instead of the CV in the EWMA statistics, (referred to as the OSE chart). In general, the performance of the OSE chart was shown to be better than the previous chart [5].

Calzada and Scariano [7] recently introduced a synthetic model to detect CV shifts. The results showed that this chart [7] is more sensitive than the Shewhart CV chart [4] but performs worse than the OSE chart. Castagliola et al. [8] presented a Shewhart-type CV procedure with variable sampling intervals. This chart showed weaker performance than the OSE chart. Later, a CV chart with supplementary run rules was proposed by Castagliola et al. [9] (referred to as the SRR chart). Based on [6], a new modified EWMA chart was designed by Zhang et al. [10] (referred to as the MOSE chart), which outperforms several competing charts in terms of the ARL. In addition, Castagliola et al. [11] suggested the use of the Shewhart-type chart in short production runs, and Amdouni et al. [12] suggested a CV scheme using the dynamic sampling model proposed by Li and Qiu [13]. You et al. [14] designed a side sensitive group run CV control model (referred to as the SSGR chart). Zhang et al. [15] presented a CV chart based on resetting the normalized observations to the target in the EWMA statistics (referred to as the RES chart).
To enhance the sensitivity of the control charts for detecting CV shifts, we propose to adopt the generally weighted moving average (GWMA) control chart, which was first proposed by Sheu and Lin [16] to detect small shifts in a process mean. Additionally, the GWMA procedure was used by Sheu and Tai [17] to monitor the variability and by Sheu and Yang [18] to monitor the median or mean. Recently, Huang [19] designed a sum of squares GWMA model to simultaneously monitor the mean and/or variability. Extensive comparisons revealed that the GWMA control charts discussed above [16-19] are more sensitive than the other related charts, especially for small to medium shifts.

In this paper, motivated by the desire to further improve the performance of charts for monitoring CV shifts, we combine the GWMA procedure with a resetting model. Shu and Jiang [20] were the pioneers who proposed the resetting technique. Li et al. [21] addressed the necessary and sufficient conditions for EWMA charts with resetting boundaries. The proposed charts, referred to as OSRG charts, based on a resetting method can overcome the problem of inertia of EWMA charts and improve the detection efficiency.

The organization of the rest of this paper is as follows. In Section 2, we propose the construction process for two one-sided OSRG charts. The implementation issues related to the proposed charts are investigated in Section 3. Section 4 is devoted to a comprehensive numerical comparison of the OSRG charts and several other powerful CV charts via simulations based on their ARL performance. Two illustrative examples are presented in Section 5. Finally, some useful conclusions are summarized in Section 6.

2 The proposed OSRG procedure

2.1 The general model of the GWMA chart

The general model of GWMA is briefly presented to better understand the construction of the OSRG chart. As noted by Sheu and Lin [16], let $q_k$ represent the probability of the occurrence of event A at the $k$th sample among a sequence of independent samples. Event B and event A are complementary and mutually exclusive. $M$ is the number of samples until the occurrence of A since the previous occurrence of
A. Let $P_k = P(M > k)$. The event $(M > k) \supset (M > (k+1))$, for $k=0,1,2,\cdots$; thus, it can be easily proven that $1 = P_0 \geq P_1 \geq \cdots$. The sequence of probabilities $\{P_k\}$ is assumed to be known. Then, we can write $p_k = P(M = k) = P_k - 1 - P_{k-1}$ at the $k$th sample, while $B$ occurs in the whole front $k - 1$ samples. Furthermore, $\sum_{m=1}^{\infty} P(M = m) = \sum_{m=1}^{\infty} P(M = m) + P_k = 1$. Therefore, when the observations $\{X_k\}$ $(k = 1, 2, \cdots)$ obey an identically independent distribution, the GWMA statistic at the $k$th sample can be obtained as

$$Y_k = P(M = 1)X_k + P(M = 2)X_{k-1} + \cdots + P(M = k)X_1 + P(M > k)Y_0,$$

where the initial value $Y_0=\mu(X)$.

To make the weights of GWMA decrease from the present period to past periods, Sheu and Lin [16] chose $P_k = q_k\alpha$, $(k=0, 1, 2, \cdots)$, where the parameters $q$ $(0 \leq q < 1)$ and $\alpha$ $(\alpha > 0)$ are constant and determined by the practitioner. The adjustment parameter $\alpha$ can slightly adjust the kurtosis of the weight function so that the GWMA model becomes more sensitive than the EWMA-type chart for monitoring process shifts. Thus,

$$Y_k = \sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{i\alpha})X_{k-i+1} + q^{k\alpha}\mu, \quad k = 1, 2, \cdots.$$

The expected value of $Y_k$ is $E(Y_k) = E(X_k) = \mu, \quad k = 1, 2, \cdots$. The variance is $Var(Y_k) = Q_k Var(X_k) = Q_k\sigma^2$, where $Q_k = \sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{i\alpha})^2, k=1,2,\cdots$. The upper and lower control limits can be constructed as

$$ucl = \mu + L\sigma \sqrt{Q_k}, \quad lcl = \mu - L\sigma \sqrt{Q_k};$$

where $L$ is a constant determined to achieve a specified in-control average run length (ARL$_0$) value. Here, the GWMA model reduces to the EWMA model when $\alpha = 1$. Above, we quote from the work of Sheu and Lin [16].

2.2 The model of the OSRG chart

Suppose that $X_k = \{X_{k1}, X_{k2}, \cdots, X_{kn}\}$, for $k = 1, 2, \cdots$, is a sample of size $n$. Each observation $X_{kj}$ is independently and identically normally distributed, i.e.,
$X_{kj} \sim N(\mu_k, \sigma_k)$. Here, the mean $\mu_k$ and the standard deviation $\sigma_k$ may vary subject only to $\gamma_k = \frac{2k}{\mu_k} = \gamma_0$, that is, the value of the CV $\gamma$ must be the specified constant $\gamma_0$ in the in-control process.

The sample CV of $X_k$ is $\hat{\gamma}_k = \frac{S_k}{\bar{X}_k}$, where the sample mean $\bar{X}_k = \frac{1}{n} \sum_{j=1}^{n} x_{kj}$ and the sample standard deviation $S_k = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_{ki} - \bar{X}_k)^2}$. The sample CV statistic is the basis of the CV control chart. The distributional properties of $\hat{\gamma}$ have been extensively studied. As Iglewicz et al. [22] pointed out, $\sqrt{n} \hat{\gamma}$ follows a noncentral $t$-distribution with $n - 1$ degrees of freedom and noncentrality parameter $\frac{\gamma_0}{\gamma}$.

From the paper proposed by Castagliola et al. [6], we determine that the CV squared $\hat{\gamma}^2$ is more efficient than the CV value $\hat{\gamma}$ for monitoring the CV shift; thus, the proposed GWMA control chart is based on $\hat{\gamma}^2$. Additionally, we adopt the resetting technique to further improve the performance of CV chart, which can ameliorate the inertia problem of the control chart.

First, we define the standardized $\hat{\gamma}^2_k$ as

$$Z_k = \frac{\hat{\gamma}^2_k - \mu_0(\hat{\gamma}^2)}{\sigma_0(\hat{\gamma}^2)} ,$$

so that $Z_k$ follows an approximately standard normal distribution. Here, approximations for $\mu_0(\hat{\gamma}^2)$ and $\sigma_0(\hat{\gamma}^2)$ are provided by Breunig [23] as

$$\mu_0(\hat{\gamma}^2) = \gamma_0^2 (1 - \frac{3\gamma_0^2}{n}) ,$$

and

$$\sigma_0(\hat{\gamma}^2) = (\gamma_0^4 \left( \frac{2}{n-1} + \frac{\gamma_0^2}{n} \right) + \frac{20}{n(n-1)} + \frac{75\gamma_0^2}{n^2}) - (\mu_0(\hat{\gamma}^2) - \gamma_0^2)^2)^{1/2} .$$

Next, we winsorize $Z_k$, and then the winsorized data are applied in a conventional GWMA model. Thus, an upward GWMA chart is defined as

$$V^{'\prime}_k = \sum_{i=1}^{k} (q^{i-1}\alpha - q^i\alpha)Z^+_{k-i+1} + q^k\alpha \mu(Z^+_0), \quad k = 1, 2, \cdots$$
where $Z^+_k = \max(0, Z_k)$, and $V'_0 = E[Z^+_k|\gamma = \gamma_0]$. The mean and variance of $Z^+_k = \max(0, Z_k)$, are given by Barr and Sherrill [24], when $Z_k \sim N(0,1)$.

$$E(Z^+_k) = \frac{1}{\sqrt{2\pi}}, \quad Var(Z^+_k) = \frac{1}{2} - \frac{1}{2\pi}.$$ 

Hence, the asymptotic mean of $V'_k$ is not 0 but $\frac{1}{\sqrt{2\pi}}$ in the in-control process. To make the mean of $V'_k$ equal to 0, the GWMA recursion in equation (5) can be rewritten as

$$V^+_k = (\mathcal{P}_0 - \mathcal{P}_1)U^+_k + (\mathcal{P}_1 - \mathcal{P}_2)U^+_{k-1} + \cdots + (\mathcal{P}_{k-1} - \mathcal{P}_k)U^+_1 + \mathcal{P}_k \mu(U^+_0)$$

where $U^+_k = Z^+_k - \frac{1}{\sqrt{2\pi}}$, $k = 1, 2, \cdots$, and $\mu(U^+_0) = 0$. The starting value of equation (6) is $V^+_0 = 0$.

The asymptotic mean and variance of equation (6) can be obtained as

$$E(V^+_k) = [(\mathcal{P}_0 - \mathcal{P}_1) + (\mathcal{P}_1 - \mathcal{P}_2) + \cdots + (\mathcal{P}_{k-1} - \mathcal{P}_k)] \mu(U^+_0)$$

$$= 0$$

and

$$Var(V^+_k) = [(\mathcal{P}_0 - \mathcal{P}_1)^2 + (\mathcal{P}_1 - \mathcal{P}_2)^2 + \cdots + (\mathcal{P}_{k-1} - \mathcal{P}_k)^2] Var(U^+_0)$$

$$= \sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{\alpha})^2 (\frac{1}{2} - \frac{1}{2\pi}).$$

In equation (8), $\sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{\alpha})^2$ was proven to be convergent as $k \to \infty$ by Chakraborty et al. [25]; thus, the variance does not diverge to infinity when the process runs for a long time.

For simplicity, denote the revised upper-sided GWMA chart in equation (6) as the upward OSRG chart. Finally, the upward OSRG chart triggers an out-of-control signal as soon as $V^+_k$ is greater than the corresponding upper control limit ($ucl_k$),

$$ucl_k = L^+ \sqrt{Var(V^+_k)}, \quad k = 1, 2, \cdots,$$

where $L^+$ is the control limit coefficient of this model. We obtain the $L^+$ value for the upward OSRG control chart via bisection searching algorithms to achieve the
desired ARL₀ value. After obtaining an appropriate value of \( L^+ \), we can calculate
the time-varying control limits in equation (9) for \( k = 1, 2, \cdots \); then, \( V^+_k \) can be
compared with the upper control limit \( ucl_k \) to obtain the run length.

Analogously, the downward OSRG control chart is defined as

\[
V^-_k = \sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{\alpha}) U^-_{k-i+1} + q^{\alpha} \mu(U^-_0), \quad k = 1, 2, \cdots
\]

where \( U^-_k = \min(0, Z_k) + \frac{1}{\sqrt{2\pi}} \) and \( \mu(U^-_0) = 0 \).

The asymptotic mean and variance of equation (10) can be written as \( E(V^-_k) = 0 \)
and \( Var(V^-_k) = \sum_{i=1}^{k} (q^{(i-1)\alpha} - q^{\alpha})^2(\frac{1}{2} - \frac{1}{2\pi}) \). Hence, set \( V^-_0 = 0 \), and the corre-
sponding lower control limit (lclₖ) is

\[
lcl_k = -L^- \sqrt{Var(V^-_k)}, \quad k = 1, 2, \cdots
\]

where \( L^- \) is a constant and can be calculated by the bisection algorithms to achieve
the given ARL₀ value, and \( Var(V^-_k) = Var(V^+_k) \).

The one-sided chart triggers an alarm as soon as \( V^+_k > ucl_k \) or \( V^-_k < lcl_k \). The
two-sided version of the OSRG chart may be a combination of two one-sided charts
for monitoring both upward and downward CV shifts; see Qiu [26]. For given \( q \) and \( \alpha \) values, the ARL value of the two-sided version is denoted as \( ARL^* \), and the ARL
values of the two one-sided charts are \( ARL^+ \) and \( ARL^- \), respectively. Then, these
values follow the approximate equation

\[
\frac{1}{ARL^*} = \frac{1}{ARL^+} + \frac{1}{ARL^-}.
\]

In fact, the one-sided GWMA charts can ameliorate the inertia of the two-sided
GWMA model, and the proposed OSRG chart introduces a resetting technique into
the one-sided GWMA chart. The performance of the OSRG chart will be presented
in the following sections.

3 Implementation issues

In this section, we first introduce a search algorithm for the control limit constants
\( L^+ \) and \( L^- \). In addition, we present a time efficiency and error analysis of this
proposed algorithm. Then, the sensitivity analysis of the proposed OSRG chart is expounded. Finally, the accuracy analysis of the proposed procedure is discussed.

A control chart cannot be implemented without numerical results for the so-called average run length (ARL). The number of samples collected from the initial time point under consideration to the occurrence of an out-of-control signal is named the run length. When the process is in control, the mean of the run length is named the in-control ARL, which is often denoted by $\text{ARL}_0$. The number of samples collected from the occurrence of a shift to the time of the signal is called the out-of-control run length, and its average value is the out-of-control ARL, which is referred to as $\text{ARL}_1$. The ideal chart is that its $\text{ARL}_0$ should be sufficiently large to avoid false alarms and $\text{ARL}_1$ should be sufficiently small to detect shifts quickly. In statistical process control, $\text{ARL}_0$ is usually fixed at a given level, and $\text{ARL}_1$ is made as small as possible. ARL values can be determined through either theoretical methodologies or computer simulation approaches; see Qiu [27] and Li et al. [28].

### 3.1 Search algorithm

This section introduces a search algorithm for the control limit constants $L^+$ and $L^-$ based on a Monte Carlo simulation and bisection method (see Qiu [29] and [30] and Li et al. [31]). For a given $\text{ARL}_0$ (denoted as $A_0$) and a given combination of $q$, $\alpha$, $\gamma_0$ and $n$, the value of $L^+$ in procedures (6)-(9) can be sought in the interval $[0, U_{L^+}]$, where $U_{L^+}$ is an upper bound satisfying the condition that the in-control ARL of the procedure is larger than the given $A_0$ when $L^+=U_{L^+}$. Let $\rho > 0$ be a small number representing the required estimation accuracy. Then, the search is performed iteratively as follows.

**Step 1.** In the $i$-th iteration, $L^+$ is sought in the range $[L^{(i)}_{L^+}, U^{(i)}_{L^+}]$. When $i=1$, $L^{(1)}_{L^+}=0$ and $U^{(1)}_{L^+}=U_{L^+}$. Set $L^{(i)}=(L^{(i)}_{L^+} + U^{(i)}_{L^+})/2$.

**Step 2.** A series of random variables is generated from the normal distribution with probability parameter $\text{CV}=\gamma_0$. These random variables are used in procedure (6), and the run length value is obtained by running procedures (6)-(9).

**Step 3.** Repeat Step 2 $M$ times, where $M$ is the Monte Carlo sample size, and obtain the run length distribution. The in-control ARL value $\text{ARL}_0^{(i)}$ of the upper-sided OSRG chart with this $L^{(i)}$ value for the given combination $(q, \alpha, \gamma_0, n)$ is
calculated by averaging all obtained run length values.

**Step 4.** If the ARL\(^{(i)}\) value computed in Step 3 is included in \([A_0-\rho, A_0+\rho]\), then the algorithm stops, and \(L^+\) is set to the \(L^{+(i)}\) value used in Step 3. Otherwise, let

\[
\begin{align*}
L_{L+}^{(i+1)} &= L^{+(i)} \quad \text{if } ARL_0^{(i)} < A_0, \\
L_{L+}^{(i+1)} &= L^{(i)} \quad \text{if } ARL_0^{(i)} > A_0, \\
L^+^{(i+1)} &= \left(\frac{L_{L+}^{(i+1)} + U_{L+}^{(i+1)}}{2}\right).
\end{align*}
\]

**Step 5.** When \(|L^{+(i+1)} - L^{+(i)}| < \varepsilon\), where \(\varepsilon > 0\) is another prespecified threshold value, the algorithm stops, and \(L^+\) is set to \(L^{+(i)}\). In such a case, a statement is presented to remind the user of the actual value of \(ARL_0^{(i+1)}\). Otherwise, the algorithm proceeds to the next iteration.

Analogously, we can obtain the \(L^-\) value in the equations (10)-(11). Based on the above algorithm, Tables 1 and 2 present the \(L^+\) and \(L^-\) values for different combinations of \((q, \alpha, \gamma_0, n)\) when \(ARL_0\) is 370.

**Tables 1 and 2 about here**

Based on equation (1), the width of the control limits of the GWMA model, which is determined by the constant \(L\) for fixed \(q \) and \(\alpha\), increases with time \(t\) and converges to the steady-state value.

Tables 1 and 2 show that the control limit constant \(L^+\) or \(L^-\) for a specified \(ARL_0\) is influenced by the parameters \(\alpha, q, n\) and \(\gamma_0\). For given values of \(\alpha, \gamma_0\) and \(n\), the \(L^+\) values decrease and, in almost all cases, the \(L^-\) values increase as \(q\) increases. Furthermore, for any given combination of \((\alpha, q, \gamma_0)\), \(L^+\) decreases and \(L^-\) increases when \(n\) increases. For any fixed combination of \((\alpha, q, n)\), \(L^+\) increases and \(L^-\) decreases when \(\gamma_0\) increases. Additionally, for any specified combination of \((q, n, \gamma_0)\), \(L^+\) decreases and \(L^-\) shows a nonuniform trend of variation as \(\alpha\) increases. In general, \(L^+\) decreases with an increase in \(q, \alpha\) or \(n\) and increases with an increase in \(\gamma_0\). By contrast, \(L^-\) decreases with an increase in \(\gamma_0\) or \(q\) and increases with an increase in \(n\).

Based on the search algorithm presented above, all results were implemented in Fortran 90 with the IMSL package. The “RNNOR” routine was used to generate standard normal random variables. The computation time strongly depends on the
preselected values \( \rho \) and \( \varepsilon \) and the total sample size \( M \). A larger (smaller) value of \( M \) and smaller (larger) values of \( \rho \) and \( \varepsilon \) will yield a more (less) accurate result, but more (less) time will be consumed. To balance the time accuracy and consumption, we recommend values of 10000, 2 and 0.003 for \( M \), \( \rho \) and \( \varepsilon \), respectively. The execution time is several minutes on an Intel Core i7-5500U 2.40 GHz CPU.

Next, to obtain error estimates for the search algorithm, let the run length be a random variable \( R \), and let the run length distribution be denoted by \( P(R = x) \), where \( x \) is a realization of \( R \). To evaluate the error of the OSRG control chart based on the search algorithm, we study the distribution of the realizations of in-control ARL values for a prespecified \( \text{ARL}_0 = \text{A}_0 \), which can be obtained from the distribution of \( R \). However, it is not easy to obtain analytical results for such a run length distribution \( P(R = x) \). Therefore, we employ Monte Carlo simulations to approximate the actual in-control ARL value distribution of the proposed OSRG control charts with the parameters obtained by the search algorithm, as described in the following procedure. For the given parameters \( q \), \( \alpha \), \( n \) and \( \gamma_0 \) and a specified \( \text{ARL}_0 \) value of 370, a corresponding control limit constant \( L \) (\( L^+ \) or \( L^- \)) is calculated using the proposed bisection search algorithms. By substituting this value of \( L \) into the corresponding OSRG control chart, we simulated 1000 realizations of the ARL value for this OSRG procedure when the process is in control. Lastly, based on the numerical results, we calculated the cumulative probability \( P(|\text{ARL}_0 - 370| \leq e) \), where \( e \) is the error value. The cumulative function curve for the error estimates is shown in Figure 1. We can see that the probability reaches 50% when \( e = 4 \) and 90% when \( e = 10 \), while the probability is 1.0 when \( e = 15 \). Because the error values are small, the algorithm is satisfactory.

3.2 Sensitivity analysis

We follow another design approach to obtain the zero-state ARL\(_1\) value in the simulation. First of all, when the process is in control (\( \gamma = \gamma_0 \)), the \( q \), \( \alpha \), \( n \) and \( L \) values are selected to yield a target zero-state ARL\(_0\). Then, based on the above \( q \), \( \alpha \), \( n \) and \( L \) values, the zero-state ARL\(_1\) is calculated for a given \( \gamma = \gamma_1 = \tau \gamma_0 \neq \gamma_0 \), where \( 0 < \tau < 1 \) corresponds to a decrease in the nominal CV, while \( \tau > 1 \) corresponds to
an increase in the nominal CV. That is,

\[ ARL_1 = ARL(\gamma_1, \gamma_0, q, \alpha, L, n), \]  

subject to the constraint

\[ ARL(\gamma_0, \gamma_0, q, \alpha, L, n) = ARL_0. \]

Based on procedures (13)-(14), the out-of-control performance characteristics of the OSRG charts in terms of the ARL\(_1\) values are shown in Tables 3 and 4. The proposed OSRG control charts have four parameters: the design parameter \(q\), the adjustment parameter \(\alpha\), the sample size \(n\) and the specified in-control CV value \(\gamma_0\). Tables 3 and 4 show that the ARL\(_1\) values are influenced by the four parameters.

Note that if \(\alpha = 1\) and \(q = 1 - \lambda\) in equation (6) or (10), then the GWMA-type control chart reduces to an EWMA-type control chart with a time-varying control limit as follows:

\[ V_k = \lambda U_k + (1 - \lambda)V_{k-1} \quad k = 1, 2, \cdots. \]

Thus, in a sense, the parameter \(q\) of the GWMA chart is equivalent to the parameter \(\lambda\) of the EWMA chart. The effect of \(q\) on ARL is consistent with the effect of \(\lambda\). The effects of various parameters on the ARL are analyzed as follows.

We choose different levels for each of the four parameters (i.e., \(\gamma_0 = 0.1\) or 0.2; \(n = 5, 10\) or 15; \(q = 0.5\) or 0.9; and \(\alpha\) is fixed to 0.75) to determine the effects of
parameters $\gamma_0$, $q$ and $n$ on the performance of the OSRG charts. It is assumed that the prespecified ARL$_0$ value is 370. Figure 2 shows the ARL values for the CV shifts under the above settings.

First, from Figure 2, by comparing the left side ($\gamma_0 = 0.1$) and the right side ($\gamma_0 = 0.2$), it can be shown that the ARL$_1$ values are generally larger when $\gamma_0 = 0.2$ in the upper-sided OSRG chart. By contrast, the ARL$_1$ values are generally larger when $\gamma_0 = 0.1$ in the lower-sided OSRG chart. Additionally, we find that when $q$ or $n$ increases, ARL$_1$ decreases in both the upward and downward OSRG charts. In other words, the larger the $q$ or $n$ value is, the more sensitive the OSRG charts are for monitoring any upward or downward shift. Considering the effect of $n$, it is obvious that the larger the value of $n$ is, the more adequate the information is that we obtain from the detected samples; thus, a larger value of $n$ can improve the sensitivity of the chart.

When $q$ is varied and $\alpha$ is fixed, Figure 3 shows the weight of each term (except the last term, $\mu(U_0^+)$ or $\mu(U_0^-)$) in the statistic $V_k^+$ or $V_k^-$ (denoted as $V_k$) in the

![Graphs showing ARL values for different parameters](image-url)
OSRG chart (when $\alpha = 0.75$). By comparing the left side ($q = 0.5$) and the right side ($q = 0.95$) of each subfigure, we can see that when a larger $q$ is chosen, a lesser weight is assigned to the current observation $U_k^+$ or $U_k^-$ (denoted as $U_k$) in equation (6) or (10), and greater weights are assigned to previous observations. However, when a smaller $q$ is chosen, a greater weight is assigned to the current observation $U_k$. In the special case of $q = 0$, $V_k = U_k$ in equation (6) or (10), the control chart based on $V_k$ is simply a Shewhart chart. In a procedure, for small or moderate shifts, even an observation far from the in-control value may not result in an immediate out-of-control signal. In such a case, the process exhibits a CV shift from $\gamma_0$ to $\gamma_1$ at time point $t$ ($1 \leq t \leq k$) that leads to changes in the actual values of equations (3) and (4); thus, the actual mean of $V_k^+$ becomes

$$
E(V_k^+, t) = [(\bar{P}_0 - \bar{P}_1) + \cdots + (\bar{P}_{k-t} - \bar{P}_{k-t+1})]\mu_1 + [(\bar{P}_{k-t+1} - \bar{P}_{k-t+2}) + \cdots + (\bar{P}_{k-1} - \bar{P}_k + \bar{P}_k]\mu_0
$$

$$
= (1 - q^{(k-t+1)\alpha})\mu_1 + q^{(k-t+1)\alpha} \mu_0
$$

$$
= (1 - q^{(k-t+1)\alpha})\mu_1,
$$

(15)

where $\mu_0$ is the in-control mean of $U_k^+$ (i.e., $\mu_0 = 0$) and $\mu_1$ is the out-of-control mean of $U_k^+$ ($k = 0, 1, \ldots$). From equation (15), we can see that $E(V_k^+, t)$ is a weighted
average of $\mu_0$ and $\mu_1$ and that the weight of $\mu_1$ increases when $k$ increases or $q$ decreases. Therefore, the larger the value of $q$ is, the smaller the value of $E(V_k^+, t)$. In such a case, the actual $E(V_k^+, t)$ value is closer to the nominal value of $E(V_k^+) = 0$, which is used in the control limit of the OSRG chart. This result also applies to the downward OSRG chart. The effect of the $q$ value on the ARL$_1$ value in the GWMA chart is consistent with that of the $\lambda$ value in the EWMA chart, as reported in the literature (see Qiu [32], for example).

Next, we observe the effect of $\alpha$ on ARL$_1$. In Figure 3, compared to when $\alpha = 1$, when $\alpha < 1$, greater weights are assigned to previous observations. However, when $\alpha > 1$, a greater weight is assigned to the current observation $V_k$. Moreover, when $q$ is large and $\alpha > 1$, the GWMA weights do not always decrease from the present observation to the more distant past observations. Therefore, we do not recommend $\alpha > 1$. Additionally, from Table 3, we can see that when $\alpha = 0.5$, the ARL$_1$ values are the largest, and consequently, the performance of the OSRG chart for monitoring any upward shift is unsatisfactory for any fixed combination of $(q, n, \gamma_0)$. Furthermore, for monitoring small or moderate upward shifts, the performances of the three OSRG charts when $\alpha = 0.75$, 0.8 or 0.9 are similar and slightly better than that when $\alpha = 1.0$. To evaluate the sensitivity of the lower-sided OSRG chart, we analyze Table 4. It is seen that when $n$ is small (e.g., $n = 5$), for any fixed combination of $(q, \gamma_0)$, ARL$_1$ decreases when $\alpha$ decreases, especially for small and moderate shifts ($0.75 < \tau \leq 1$). In such a case, the ARL$_1$ values are smallest when $\alpha = 0.5$. However, as $n$ increases (e.g., $n = 10$ and 15) and for large values of $q$ ($0.95 \leq q < 1$), the ARL$_1$ values are smaller when $\alpha = 0.75$ or 0.8 than when $\alpha = 0.5$ or 1.0. It is reasonable to conclude that the ARL values are influenced by $\alpha$ for the OSRG charts, which is consistent with previous results in the literature, such as those of Sheu and Lin [16].

In general, the weights in the OSRG charts are predominantly influenced by $q$ and $n$; consequently, the effect of $\alpha$ on ARL$_1$ is weaker than that of $q$ or $n$. A practitioner can choose $n$ as large as possible, a sufficiently large $q$ value ($q = 0.95$ is satisfactory) and $\alpha = 0.75$ or 0.8 to rapidly detect any upward or downward CV shift.
3.3 Robustness analysis

The robustness of the OSRG chart for nonnormal distributions is studied. For example, for the upper-sided OSRG chart with $\gamma_0 = 0.1$, $\alpha = 0.8$, $q = 0.95$, $n = 5$ and $\text{ARL}_0 = 370$, the corresponding $L^+ = 3.44$ (from Table 1), and a random variable following t-distribution (degrees of freedom $n < 500$), the monitoring ability of this OSRG chart is satisfactory for an upward shift $1.2 \leq \tau \leq 2$ (from Figure 4). Additionally, the sensitivity of the OSRG chart for detecting CV shifts increases with increasing degrees of freedom of the t-distribution, although the in-control ARL values are greater than 370. In the above OSRG chart for random numbers following a lognormal distribution, the monitoring ability is obviously weaker than that for the t-distribution. When the random numbers follow a Gamma distribution (shape parameter $\alpha \leq 1$, scale parameter $\beta = 1$), the false alarm probability of the OSRG chart is greatly increased in Phase I. Hence, the OSRG chart can be used under a t-distribution to detect moderate and large CV shifts; however, the proposed chart is not applicable for a biased distribution.

4 Comparisons

The proposed OSRG control charts are robust means of detecting various shifts in the CV, as seen from the above numerical analysis of $\text{ARL}_1$ (from Tables 3 and 4). We will further verify this result. Usually, a control chart outperforms its competitors if it has a smaller $\text{ARL}_1$ for a shift of a given size and a specified value of $\text{ARL}_0$. In
this section, the performance of the proposed OSRG control charts is compared with those of a number of competing CV charts for the same value $\text{ARL}_0 = 370$.

### 4.1 Comparison with the RES, MOSE and OSE charts

Here, the one-sided OSRG charts are compared with three EWMA CV control models, namely, the RES, MOSE and OSE control charts. The upper-sided RES chart is defined as follows:

$$W^+_k = \lambda U^+_k + (1 - \lambda)W^+_k - 1.$$  

The corresponding upper control limit is calculated as

$$ucl = K^+ \sqrt{\frac{\lambda}{2 - \lambda}} \sigma_{U^+_k},$$

where $K^+$ is chosen to achieve a specified $\text{ARL}_0$. The starting value $W_0 = 0$.

The lower-sided RES chart is defined as

$$W^-_k = \lambda U^-_k + (1 - \lambda)W^-_{k-1}.$$  

The corresponding lower control limit is calculated as

$$lcl = -K^- \sqrt{\frac{\lambda}{2 - \lambda}} \sigma_{U^-_k},$$

where $K^-$ is chosen to achieve a specified $\text{ARL}_0$ and $\sigma_{U^+_k} = \sigma_{U^-_k} = \sqrt{\frac{1}{2} - \frac{1}{2\pi}}$.

It is clear that the proposed OSRG procedure (equations (6)-(11)) is different from the RES procedure (equations (16)-(19)). First, the OSRG charts are based on the GWMA model, while the RES procedure is based on the EWMA model; this is the most significant difference between the two models. Second, the OSRG charts have time-varying control limits that are affected by two parameters, $\alpha$ and $q$, while the algorithms for the RES charts use fixed-width control limits that depend on a single parameter, $k$. Moreover, the variance of the statistic $V^+_k$ (or $V^-_k$) in the OSRG chart is different from that of the statistic $W^+_k$ (or $W^-_k$) in the RES chart. Additionally, the sensitivity of the RES chart is related to the magnitude of a shift $\tau$ when $\gamma_0$ is fixed. Thus, for a given shift $\tau^*$, the optimal pair $(\lambda^*, K^*)$ that is obtained
by the optimization algorithm in [15] yields the smallest possible $\text{ARL}_1$ in the RES charts. By contrast, the selection of the parameters $q$ and $\alpha$ of the OSRG charts does not depend on the magnitude of the CV shift $\tau$, so we can apply only the proposed search algorithm to obtain the numerical results.

The differences among the three EWMA-type control charts listed above for monitoring CV shifts are described in [15]. Numerical comparisons of the OSRG charts with the three EWMA-type CV charts are presented as follows. In the OSRG charts, the parameter combination $(q, \alpha)$ is specified to be $(0.95, 0.75)$, which corresponds to the optimal RES, OSE and MOSE control charts. The simulation results are presented in Figure 5. This figure shows that the OSRG chart outperforms all three competing charts for all upper-sided shift sizes $\tau = \{1.1, 1.25, 1.5, 2.0\}$ and all lower-sided shift sizes $\tau = \{0.5, 0.65, 0.8, 0.9\}$, regardless of the values of $n$ and $\gamma_0$. For instance, for $(n, \tau, \gamma_0)=(5, 1.1, 0.1)$, the $\text{ARL}_1$ of the OSRG model is 35.9, which is nearly 23% less than the value of 46.5 for the RES chart, 20% less than the value of
44.5 for the MOSE chart and 30% less than the value of 51.5 for the OSE model. As
the shift value becomes larger, the advantage of the OSRG procedure increases. In
such a case of \( \tau = 1.5 \), the ARL\(_{\tau} \) of the OSRG chart is 3.5, which is approximately
35% less than the value of 5.4 for the RES and MOSE charts and 40% less than
the value of 5.8 for the OSE chart. For the decreasing case, for instance, when \((n, \tau, \gamma_0) = (5, 0.9, 0.1)\), the ARL\(_{\tau} \) value of the OSRG model is 31.6, which is nearly 38%
less than the value of 51.3 for the RES chart, 27% less than the value of 43.4 for
the MOSE chart and 41% less than the value of 54.1 for the OSE chart. Therefore,
based on the ARL\(_{\tau} \) values, the two one-sided OSRG charts are more effective than
the RES, OSE and MOSE charts.

Moreover, we use the relative mean index (rmi) values to assess the overall per-
formances of the charts over a range of shifts. This metric is defined as

\[
\text{rmi} = \frac{1}{N} \sum_{i=1}^{N} \frac{ARL_{(\tau_i)} - \text{MARL}_{(\tau_i)}}{\text{MARL}_{(\tau_i)}},
\]

where \( N \) is the total number of parameter shifts considered. For monitoring a shift \( \tau_i \),
ARL\(_{(\tau_i)} \) represents the ARL\(_{\tau} \) value of the given chart, and \( \text{MARL}_{(\tau_i)} \) is the smallest
ARL\(_{\tau} \) value of all of the compared charts. The smaller the relative mean index is,
the better the performance of the chart is. As seen from Figure 5, the relative mean
index values indicate that the OSRG charts outperform the RES, OSE and MOSE
charts. Hence, the proposed charts offer improved sensitivity compared with the
EWMA-type charts for monitoring CV shifts.

### 4.2 Comparison with the SRR and SSGR charts

Here, we compare the OSRG charts with two Shewhart-type charts, namely, the
SRR and SSGR charts. According to Castagliola et al. [9], in most cases, the 4-
out-of-5 SRR chart exhibits good performance; therefore, we choose this chart as a
benchmark. The SRR chart is two-sided, so we compare it with the combination
of the two one-sided OSRG charts, where the parameters are set to \( q = 0.95 \) and
\( \alpha = 0.75 \), for detecting both increasing and decreasing shifts in the CV. To ensure a
fair comparison between the SRR and OSRG charts, according to equation (12), the
ARL\(_0 \) value of each of the one-sided OSRG charts should be approximately 740, so
that the ARL$_0$ value of the combined charts will be 370. The simulation results are reported in Figure 6. The ARL$_1$ of the OSRG model is usually much smaller than those of the SSGR and SRR charts for various shift sizes. For instance, for detecting upward shifts, with $\tau = 1.1$, $\gamma_0 = 0.1$ and $n = 10$, the ARL$_1$ of the OSRG chart is 21.0, which is approximately 76% less than the value of 87.7 for the SRR chart and 60% less than the value of 52.0 for the SSGR chart. For detecting downward shifts, with $\tau = 0.8$, $\gamma_0 = 0.2$ and $n = 5$, the ARL$_1$ of the OSRG model is 6.1, 93% less than the value of 83.0 for the SRR chart and 96% less than the value of 162.0 for the SSGR chart. The relative mean index values thus indicate that the OSRG chart outperforms the SRR and SSGR charts.

In general, the OSRG scheme performs substantially better than the existing EWMA and Shewhart CV charts, especially for detecting small or moderate shifts in the process CV.
5 Two real data examples

The first numerical example is obtained from Castagliola et al. [6] for a sintering process. The process manufactures the parts required to guarantee a pressure test drop time $T_{pd}$. A preliminary regression study of the pressure drop time $T_{pd}$ demonstrated that the standard deviation of $T_{pd}$ is proportional to its mean, i.e., the CV $\gamma_{pd} = \sigma_{pd}/\mu_{pd}$ is constant.

Based on the data of Phase I, we can obtain an estimate for the in-control CV value $\gamma_0 = 0.417$. For consistency with [6], we set the shift magnitude $\tau$ to 1.25. Table 3 shows that based on ARL$_1$, the properties of the upward OSRG chart with $(q = 0.95, \alpha = 0.8)$ surpass those of charts with other parameter combinations for small shifts (i.e., $\tau \leq 1.3$). Thus, for detecting this shift, when $n = 5$, we set the parameters to $(q = 0.95, \alpha = 0.8)$. The search algorithm presented above (in Section 3.1) can be used to obtain the control limit parameter $L^+ = 5.929$ when the ARL$_0$ value is 370.

The dataset of Phase II consists of 20 new samples (refer to Table 7 of [6]) obtained from the process after the occurrence of a particular factor that increased the process variability.

The upward OSRG chart is plotted in Figure 7 (left side). This figure shows
that the OSRG chart yields an out-of-control signal for the first time at sample 3, a second time at sample 7, and again after the 13th sample. The chart statistics for all subsequent samples (except the 18th sample) are above the control limit. Note that the RES, OSE and MOSE charts detect the first out-of-control signal at the 10th, 13th and 13th samples, respectively.

Referring to the actual data in Table 7 of [6], we see the CV value of sample 3 is 0.932, which is 123.5% greater than the in-control CV value of 0.417, which implies an upward shift of 123.5% in the CV. The CV value of sample 7 is 1.058, which is an upward shift of 153.7% in the CV. These are two very obvious out-of-control assignable sources, which are sensitively acquired and triggered by the OSRG chart. However, the RES chart triggers the first out-of-control signal at sample 10 ($\gamma = 0.662$), which is 7 runs later than that using the OSRG chart. Additionally, the observation of sample 18 does not exceed the upper control limit in the OSRG chart, which implies that the 18th sample is in control. This result is consistent with the actual value of the 18th sample of 0.425, because the in-control interval is from $\gamma_0 = 0.417$ to $\gamma_1 = \gamma_0 \times \tau = 0.417 \times 1.25 = 0.521$. The in-control characteristics of sample 18 are not presented in the RES chart. Therefore, our OSRG chart shows more accurate information for the observations than the RES chart and a superior detection ability.

The following actual data from a spinning process were kindly provided by a Chinese fiber company that manufactures the 154dtex/36f type of nylon 6 yarn. To guarantee the stability and reliability of the product quality, manufacturers pay close attention to the CV value of the elongation of the nylon yarn. This CV value is an important physical index of fiber products and influences the grade of the nylon yarn and the quality of the subsequent processing. If the other process parameters are mismatched, the CV value of the elongation of the nylon yarn will be greatly increased. Thus, based on monitoring the CV value of the elongation, the manufacturer undertakes the coordination of the spinning technology and equipment to improve the product quality.

[Insert Table 5 about here]

We present an elongation dataset of 20 samples of size $n = 30$ (see the left side of Table 5) when the production process is in control (Phase I); their regression
results demonstrate that the ratio between the standard deviation and the mean of
the elongation $X$ (in %) of the 154dtex/36f-type nylon 6 yarn is constant, i.e., the
elongation CV value $\gamma_X = \sigma_X / \mu_X = \gamma_0$. Here, based on the data of Phase I, the
in-control CV value can be estimated to be $\hat{\gamma}_0 = \frac{1}{20} \sum_{k=1}^{20} \hat{\gamma}_k = 0.1195$.

To detect an increase in the CV value of the elongation, we assume the shift
magnitude $\tau = 1.15$; this is a deterioration in the production. A set of parameters
of the upper-sided OSRG procedures (6) to (9) is applied: $(q = 0.95, \alpha = 0.8, L^+ = 2.972)$ when $\text{ARL}_0=370$. The out-of-control dataset of 20 samples, each of size 30,
collected during Phase II is presented in Table 5 (right side). From Table 5, based
on the in-control CV value $\gamma_0 = 0.1195$, the upward shifts in the CV values of 20
samples are all less than 26.8%. This means that the shift magnitude is small, so
the out-of-control assignable sources of the control charts are more difficult to detect
than when monitoring large shifts. Figure 8 (left side) displays the 20 corresponding
statistics $V^+$ and the time-varying upper control limits of the OSRG chart to monitor
the CV variation of the elongation of nylon 6 yarn. It is shown that an out-of-control
signal is given at sample 16 in the OSRG chart. In fact, the out-of-control signal
from the 16th sample onward is indicative of an upward shift in the process CV.

As a comparison, the RES chart with an optimal parameter pair ($\lambda^* = 0.05$,
$K^* = 2.707$) presents a plot of the control statistics with the upper-sided control limit $\text{ucl} = 0.2531$ in Figure 8 (right side). We can see that the first out-of-control signal is triggered at sample 17, which is 1 run later than that from the OSRG chart. Referring to the actual data of Phase II in Table 5, we see the CV value of sample 16 is 0.1515, which is 26.78% greater than the in-control CV value of 0.1195. The CV value of sample 17 is 0.1367, which is an upward shift of 14.39% in the CV. Therefore, the 16th sample, which is missed and not triggered by the RES chart, is more obvious out-of-control assignable source than sample 17. The OSE and MOSE charts also show the first out-of-control signal at the 17th sample. These results again indicate that the OSRG chart can detect CV shifts more quickly than the other completing charts.

6 Conclusions

This paper provides two one-sided OSRG control charts for monitoring upward and downward CV shifts. The proposed scheme is designed by truncating negative normalized observations to zero in the GWMA statistic. Tables of the control limit parameters $L^+$ and $L^-$ corresponding to various values of $q$, $\alpha$, $n$ and $\gamma_0$ are provided and can be used as a design aid. In practice, when there is no foreknowledge of shift patterns, $0.9 \leq q \leq 0.95$ and $0.75 \leq \alpha \leq 0.9$ are recommended for detecting small to large shifts. Both the in-control and out-of-control performances of the OSRG chart are studied in terms of the ARL. The performance of the proposed chart is compared with those of several powerful CV charts for monitoring various upward and downward shifts by means of extensive simulation studies. Based on the numerical results, the overall conclusion is that the OSRG CV chart is superior to all other compared charts, although it performs similarly to the SSGR chart for large shifts. When the shift size is smaller, the advantage of the OSRG chart is greater. A definite improvement in the monitoring of the CV shifts is achieved by the proposed OSRG chart. Hence, the proposed OSRG chart has the following good features. First, no additional parameters are required except for the design parameter $q$, the adjustment parameter $\alpha$ and the control limit constant $L$; thus, the chart can easily be designed and constructed for various applications. Second, owing to the good properties of the adjusted time-varying control limits, the proposed
chart is more sensitive for detecting start-up problems and small shifts than a chart with fixed control limits. Third, the resetting technique adopted in the OSRG chart can ameliorate the inertia problem to improve the efficiency of the control chart. Additionally, based on an error analysis, the errors of the bisection algorithms for searching the $L$ value are small. These findings show that the OSRG chart is a useful tool for practitioners due to its ability to detect various types of CV shifts.

Note that the OSRG and other existing CV procedures aim to monitor a univariate CV shift in a normal process. The detection of multivariate CV shifts has received very little attention in the existing literature (see Yeong et al. [33]). However, in many situations, multiple characteristics need to be considered and monitored; therefore, detecting the multivariate CV poses a challenge to practitioners. This challenge represents our next research direction.

Acknowledgements

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References


Table 1: The control limit constant $L^+$ of the upward OSRG control charts when $\gamma_0 = 0.1, 0.2$, $ARL_0 = 370$ and $n = 5, 10, 15.$

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Table 2: The control limit constant $L^-$ of the downward OSRG control charts when $\gamma_0 = 0.1, 0.2$, $ARL_0 = 370$ and $n = 5, 10, 15.$

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Table 3: The ARL values of the upward OSRG control charts when $\gamma_0 = 0.1, 0.2$, $ARL_0 = 370$ and $n = 5, 15.$

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27
Table 4: The ARL values of the downward OSRG control charts when $\gamma_0 =0.1, 0.2$, $ARL_0 = 370$ and $n = 5,15$.

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Table 5: The datasets of elongation of the 154dtex/36f type of the nylon 6 yarn from the spinning process.

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