Rank Based EWMA Procedure for Sequentially Detecting Changes of Process Location and Variability

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Abstract

This paper presents a study of a new procedure, which is based on integrating a powerful nonparametric test for the two-sample problem and EWMA control scheme to on-line sequential monitoring. The proposed procedure, based on individual observation per sample, can be used to monitor the location and the scale parameters of a univariate continuous distribution, simultaneously. An iterative computation procedure is developed for computing the monitoring statistics. A search algorithm for the control limit based on Monte-Carlo simulation and bisection method is derived and a table is provided. The sensitivity analysis on the procedure is studied in detail. Monte-Carlo simulation results show that the proposed procedure is quite robust to non-normally distributed data, and moreover, it is efficient in detecting various process shifts. A real data example from a chemical reaction process is shown to illustrate the application of our proposed procedure.

Keywords: Control Charts; EWMA; Nonparametric; Search Algorithm; Statistical Process Monitoring.

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1 Introduction

Control charts have been widely used in many areas, not only in industry, but also in areas such as transactional and service businesses, signal and image processing ([30]), health care and public health surveillance ([45]), and so forth. In addition to standard text book such as [31], interested readers could also refer to [35] for recent development on statistical process control (SPC) and to [49] for control charts for high-quality processes. Application of SPC charts involves two stages, i.e., Phase I and Phase II. In Phase I, a set of process data is collected and analyzed (retrospective analysis) all at once to determine whether the process has been in control. After Phase I, we have a "clean" set of process data that is representative of in-control (IC) process performance. In Phase II, control chart is used to monitor the process by comparing the sample statistic to the control limits for each successive sample as it is collected from the process. Performance of a Phase II SPC procedure is usually measured by the average run length (ARL), which is the average number of observations that must be collected before an observation indicates an out-of-control (OC) condition ([31]). This paper focuses on Phase II monitoring of univariate processes in cases when process observations are not normally distributed, i.e., there is lack of knowledge about the underlying distribution.

In the literature, many Phase II SPC charts have been proposed, including different versions of the Shewhart chart, the cumulative sum (CUSUM) chart, the exponentially weighted moving average (EWMA) chart, and the chart based on change-point detection (cf., e.g., [23, 31, 35]). Many of these control charts are based on the assumption that the observations of a process follow a particular probability distribution, usually normal distribution. In practice, the process observations may, however, not follow the specified (e.g. normal) distributions. In such cases, [37, 38] show that results from the charts mentioned above would be unreliable, i.e., the actual IC ARL (denoted as ARL_0) values of the conventional CUSUM chart ([34]) are much smaller than the nominal ARL_0 values when the true process distribution is t or χ^2, especially with small degree-of-freedom, which implies that the related process would be stopped too often by the control chart and consequently a considerable amount of time and resource would be wasted in such cases.

It is necessary to develop appropriate control procedures that do not require the normal distribution assumption in cases when the process distribution is actually non-normal. To this end, a number of distribution-free or

nonparametric SPC (NSPC) charts have been developed. [10, 6] give thorough overviews on existing research in the area of univariate NSPC. [47, 48] also encourage research of nonparametric methods. For some recent development, for Phase I, see, for instance, [5] propose distribution-free procedure for univariate observations based on recursive segmentation and permutation. [26, 25] propose distribution-free control charts for subgroup location and scale, respectively. [42, 43] construct a standard deviation and $\bar{X}$ control chart derived from the trimmed mean, respectively. [44, 53] propose robust estimators for location. [33] study robustness of control charts for monitoring process mean. For Phase II, to name a few, [2] propose an extrema chart based on the extrema of a sample of observations. [28] propose two nonparametric analogs of the CUSUM and EWMA control charts based on the Wilcoxon rank-sum test for detecting process mean shifts. [50] propose a non-parametric CUSUM mean chart. [39] investigate the performance of two CUSUM chart combinations that can be made to be robust to non-normality. By integrating the omnibus Kolmogorov-Smirnov and Cramer-von-Mises tests into the widely researched change-point model framework, [40] present two distribution-free charts that can detect arbitrary changes to the process distribution. [22] develop a nonparametric tool based on the change-point model. [7, 8] propose control charts based on runs and precedence statistics with runs-type signaling rules, respectively. [16, 15] propose nonparametric charts for variability and location, respectively. [20, 21] propose nonparametric EWMA charts for monitoring location based on signed-rank and precedence test, respectively. [19] develop a bivariate signed-rank test for location parameters. [51] develop a multivariate sign EWMA control chart for location parameters.

Note that most of the procedures above are only designed to detect a single shift in the location (e.g., mean) or scale (e.g., standard deviation) of the process. Generally, the process location and variability may vary simultaneously during the monitoring period. Therefore, it is desirable to construct a procedure that can not only detect changes in the process location, but also is sensitive to the shifts in the process variability. [12, 29] give overviews of the control charts in an effort to use only one chart to simultaneously monitor both process location and scale. There are also some attempts for single charts in NSPC. Among others, [52] develop a distribution-free control chart based on the integration of a powerful nonparametric goodness-of-fit test and the EWMA control scheme. [41] propose a Lepage-type hypothesis test statistic based on Mann-Whitney test statistic for monitoring process

location and Mood test statistic for monitoring process variability. Combining two popular nonparametric test statistics: the Wilcoxon rank sum test for location and the Ansari-Bradley test for scale, [32] propose a single distribution-free Shewhart-type chart. [13] propose a distribution-free Shewhart-type chart based on the Cucconi statistic. Recently, [14] extend the Shewhart-type chart of [32] to CUSUM-type chart. [1] develop a nonparametric CUSUM by utilizing a smooth bootstrap algorithm along with an adaptive nonparametric kennel density estimator. The research, however, is still demanding for nonparametric single charts.

The purpose of this paper is to propose a single NSPC procedure that can monitor the location and the scale parameters of a univariate continuous distribution, simultaneously. Most existing NSPC charts mentioned above are based on ordering or ranking information of the observations obtained at the same or different time points. Some of them require multiple observations at each time point (e.g., [20, 21]). In many industrial applications ([31], Chapter 6), due to practical and cost concerns, it may be impossible to collect more than one observation per sample. Intuitively, it would lose much information if we only use the ordering or ranking information in the observed data for process monitoring. Motivated by works of [24] and [17], we propose a single NSPC procedure that is based on individual observation and can monitor the location and the scale parameters of a univariate continuous distribution, simultaneously. The proposed procedure is compared with some competing methods in the literature, including the nonparametric likelihood-ratio EWMA (NLE) procedure of [52], the robust EWMA (RoE) of [3], and the control chart based on precedence statistics with runs-type signaling rules of [8] (denoted as Pre).

The rest part of the paper is organized as follows. Our proposed procedure is described in Section 2. Then some implementation issues are described in detail in Section 3, including iterative computation of the monitoring statistics, search algorithm for finding control limit and sensitivity analysis of the effect of the parameters on the performance. A comparison study based on Monte-Carlo simulation to evaluate the performance in comparison with some competing procedures is presented in Section 4. An application is discussed in Section 5 to demonstrate the application of the proposed method in a real world setting. Some remarks conclude this paper in Section 6. The simulation procedure by which the ARL values are derived is deferred to Appendix.

2 Proposed NSPC Procedure

Motivated by the discussions above, a new procedure is described here. We first give the assumptions and models of the proposed procedure, and then present the methodology for constructing our NSPC procedure, named RE procedure.

Assume that $X_1, X_2, \ldots, X_m$ are a sequence of independent observations obtained during Phase I process monitoring and $Y_1, Y_2, \ldots, Y_\tau, Y_{\tau+1}, \ldots, Y_n$ are a sequence of independent observations obtained during Phase II process monitoring. The cumulative distribution functions (cdf) of $X_1, X_2, \ldots, X_m$, $Y_1, Y_2, \ldots, Y_\tau$ are the same to be $F(\cdot)$ up to an unknown time point τ, and change to another cdf $G(\cdot)$ after the time point τ for $Y_{\tau+1}, \ldots, Y_n$.

As some existing NSPC procedures (e.g., [40, 28]), we assume the IC cdf $F(\cdot)$ is unknown. As a matter of fact, we assume that an IC dataset of size m has been collected in the Phase I analysis, and it can be used for estimating certain IC parameters or the IC distribution $F(\cdot)$. The set of IC Phase I sample $X_1, X_2, \ldots, X_m$ is used to represent the IC process distribution $F(\cdot)$. Note that, in practice, it might still be an important issue to do Phase I analysis efficiently in cases when $F(\cdot)$ is nonparametric and unknown, although Phase I analysis in such cases is not the focus of this paper. [26] propose a distribution-free method for defining the IC state of a process and identifying an IC reference sample. [9, 27] give comprehensive reviews of Phase I analysis, which is a broad area of quality monitoring research and has been studied by many researchers. The proposed procedure in this paper is not applicable in Phase I. And it will also be shown in Section 3.2 that the size m of Phase I sample has great effect on the IC performance on our proposed procedure.

Now let $R_i, 1 \le i \le m$, be the rank of X_i in the pooled sample $\{X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_\tau, Y_{\tau+1}, \ldots, Y_n\}$. Analogously we denote $R_i, m+1 \le i \le N$, as the rank of Y_i in $\{X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_\tau, Y_{\tau+1}, \ldots, Y_n\}$, where $N = m + n$. To design a procedure that is able to monitor the location and the scale parameters of a univariate continuous distribution, a combined statistic is usually employed, such as the Lepage statistic proposed by [32, 14], which is the sum of squares of standardized of Wilcoxon rank sum statistic Q_1 and Ansari-Bradley statistic Q_2,

$$Q_1 = \sum_{i=1}^{N} R_i I_i, \quad Q_2 = \sum_{i=1}^{N} |R_i - \frac{N+1}{2}| I_i, \tag{2.1}$$

and the Cucconi test statistic proposed by [13], which is a combination of standardized of sum of the squares of the ranks P_1 and sum of the squares of the anti-ranks P_2,

$$P_1 = \sum_{i=1}^{N} R_i^2 I_i, \quad P_2 = \sum_{i=1}^{N} (N + 1 - R_i)^2 I_i, \tag{2.2}$$

where I_i is an indicator variable with $I_i = 1$ when the i^{th} order statistic of the combined N observations is a Y; otherwise, $I_i = 0$. Note that $R_i I_i$ is the ranks of Y and $\frac{N+1}{2}$ is the average of all ranks. It seems that the difference of the ranks of Y and the average of all ranks will help detect the location and variability change of a process, if any. In Eq. (2.1) and (2.2), the ranks of X are not evidently included, which motivates us to employ the squared difference of the average of the ranks of X and Y, i.e., a statistic like

$$\left(\frac{1}{m} \sum_{i=1}^{m} R_i - \frac{1}{n} \sum_{i=m+1}^{N} R_i \right)^2. \tag{2.3}$$

We found that, the statistic in Eq. (2.3) is a special case of the statistic T_k proposed by [24] and [17], which is defined by

$$T_k = \sum_{j=1}^{k} \Big\{ \sum_{i=1}^{N} c_{Ni} b_j(\frac{R_i - 0.5}{N}) \Big\}^2, \tag{2.4}$$

where $b_j(\cdot)$ denotes the j^{th} orthonormal Legendre polynomial,

$$c_{Ni} = \sqrt{\frac{mn}{N}} \begin{cases} m^{-1} & \text{as} \quad 1 \le i \le m, \\ -n^{-1} & \text{as} \quad m+1 \le i \le N. \end{cases}$$

The Legendre polynomial $P_n(x)$ is defined recursively by

$$P_0(x) = 1, P_1(x) = x$$

and

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x).$$

The first few orthonormal Legendre polynomials $b_j(\cdot)$ are $b_0(x) = \sqrt{1/2}$, $b_1(x) = \sqrt{3/2}x$, $b_2(x) = \sqrt{5/8}(3x^2 - 1)$, and etc.

[24] and [17] proposed the statistics T_k in Eq. (2.4) for the following testing problem,

$$H_0 : F(x) = G(x) \quad \text{versus} \quad H_1 : F(x) \neq G(x),$$

and they derived the limiting distribution of T_k and its consistency under mild conditions, and also showed the efficiency of T_k through extensive simulation studies. The k values are suggested to be a small positive integer, such as 1 and 2.

Based on the consideration of Eq. (2.3) and Eq. (2.4), we construct our NSPC procedure with T_1, which turns out to be the squared difference of the average of the ranks of X and Y. One may also employ other statistic, e.g., T_2. We, here, just use T_1 to show the idea of this paper, and it can be easily generalized to T_2. Although T_1 is a function of the ranks of all observations up to time point N, it may not be effective to detect persistent changes in the process. It is well known that EWMA is more effective for small changes of process ([20, 21, 28, 52]), we suggest the following EWMA procedure at time point n,

$$\begin{cases} RE_0 = 0, \\ RE_n = (1-\lambda)RE_{n-1} + \lambda T_1, \text{ for } n \geq 1, \end{cases} \tag{2.5}$$

where $0 < \lambda \leq 1$ is the smoothing constant. An OC signal is issued if $RE_n > h$, where h is the control limit for a prespecified ARL_0. For the rest of this paper, we denote our proposed rank-based EWMA procedure as RE procedure.

Note that our proposed RE procedure calls for calculating T_1 as N increases. That is, regardless of how big N might be, one still has to calculate T_1 for small N values. We do not recommend to use the asymptotic distribution χ^2_1 derived by [24] in practice to find the control limits. Instead, we find the control limits through Monte-Carlo simulation based on a search algorithm (details deferred to Section 3.2).

3 Implementation Issues

We describe some implementation issues of our proposed RE procedure. In Section 3.1, the computation aspects are described, where we give an iterative algorithm for computing the monitoring statistics. Then, in Section 3.2,

we show a search algorithm for the control limit h based on Monte-Carlo simulation and bisection method and study the effect of Phase I sample size m on the IC performance. Finally, in Section 3.3, the sensitivity analysis of effects of parameters on ARL values of proposed procedure is discussed.

3.1 Iterative computation

It seems necessary to sort the observations $\{X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_\tau, Y_{\tau+1}, \ldots, Y_n\}$ and obtain the corresponding ranks every time when we get a new observation Y_n at time point n, which will inevitably cause a tedious computation and a heavy storage task as n gets large. Fortunately, we can make the computation much easier using the following iterative algorithm.

Note that

$$\begin{aligned} T_1 &= \left\{ \sum_{i=1}^{N} c_{Ni} b_1 (\frac{R_i - 0.5}{N}) \right\}^2 \\ &= \left\{ \sum_{i=1}^{m} \sqrt{\frac{mn}{N}} \cdot \frac{1}{m} \cdot \sqrt{\frac{3}{2}} (\frac{R_i - 0.5}{N}) - \sum_{i=m+1}^{N} \sqrt{\frac{mn}{N}} \cdot \frac{1}{n} \cdot \sqrt{\frac{3}{2}} (\frac{R_i - 0.5}{N}) \right\}^2 \\ &= \frac{3mn}{2N^3} \left(\frac{1}{m} \sum_{i=1}^{m} (R_i - 0.5) - \frac{1}{n} \sum_{i=m+1}^{N} (R_i - 0.5) \right)^2 \\ &= \frac{3mn}{2N^3} \left(\frac{1}{m} \sum_{i=1}^{m} R_i - \frac{1}{n} \sum_{i=m+1}^{N} R_i \right)^2. \end{aligned}$$

Let $X_{(1)}, X_{(2)}, \ldots, X_{(m)}$ be the order statistics of $X_1, X_2, \ldots, X_m$. Before we start Phase II monitoring, i.e., $n = 0$, the ranks of $X_{(1)}, X_{(2)}, \ldots, X_{(m)}$ are $1, 2, \ldots, m$, respectively. Then at any time point $n (n \geq 1)$ when we get the sample Y_n, we compare Y_n with $X_{(1)}, X_{(2)}, \ldots, X_{(m)}$. If $Y_n < X_{(i)}$, the rank of $X_{(i)}$ is updated by adding one; otherwise, the rank of $X_{(i)}$ remains the same as the original one. Note that the following equation holds

$$\sum_{i=1}^{m} R_i + \sum_{i=m+1}^{N} R_i = N(N+1)/2. \tag{3.6}$$

So when we obtain the ranks of $X_{(i)}, i = 1, 2, \ldots, m$ at any time point n, we can obtain $\sum_{i=1}^{m} R_i$, and then obtain $\sum_{i=m+1}^{N} R_i$ from Eq. (3.6). The

entire computation involves only order m (which is fixed and not growing) computations for any time point n, and thus the computational task can be easily managed.

3.2 Search algorithm

A search algorithm for the control limit h is based on Monte-Carlo simulation and bisection method. The Fortran program used is available from the authors upon request. A user can also code his own computer program by the guidelines below for finding h.

Step 1: Input parameters to be specified by the practitioners, including ARL_0, Phase I sample size m and smoothing constant λ.

Step 2: Set the maximum of h to, e.g., $h_{max} = 0.1$ (such that the ARL value is larger than ARL_0) and the minimum of h to $h_{min} = 0$ and set $h = (h_{max} + h_{min})/2$.

Step 3: Set $RE_0 = 0$ and $RL = 0$. Generate standard normal random variables, update RL to $RL = RL + 1$ and compute T_1 to obtain RE_n until $RE_n > h$.

Step 4: Repeat Step 3 B times, which is the Monte Carlo sample size and obtain the average of the RL, i.e., ARL. If $ARL > ARL_0$, update h_{max} to h; otherwise, update h_{min} to h.

Step 5: The algorithm is not stopped until the absolute difference of ARL and ARL_0 is less than a prefixed value ε_1 or the absolute difference of the current h and the h in the last iteration is less than another prefixed value ε_2.

Based on the search algorithm above, all the results are implemented in Fortran 95 with IMSL package. Routine "rnnor" is used to generate standard normal random variables. Note that the proposed RE procedure is nonparametric and hence the ARL values are nearly the same for other non-normal continuous distributions, such as $t(3)$ and χ^2_3 used in Section 3.3, which can be generated by routine "rnstt" and "rnchi", respectively. The computation time depends highly on the total Monte Carlo sample size B, the prefixed values ε_1 and ε_2. Hence larger value of B and smaller values of ε_1 and ε_2

will lead the result with higher accuracy, but more time consuming, and vice versa. To balance the time and accuracy, we recommend that $B = 5000$, $\varepsilon_1 = 1$ and $\varepsilon_2 = 10^{-4}$. The execution time is about several minutes on a Pentium 4 with CPU processor 3.00 GHz.

Numerical computations in Fortran software, based on the search algorithm above, are used to determine h. The results, which are displayed in Table 1, show a pretty stable and meaningful estimates of the ARL_0. We have chosen $m = 20, 50, 100, 200$ and 500 for the reference sample size and $\lambda = 0.05, 0.10, 0.20$ and 1.00 for the smoothing constant. The h values in Table 1 are for $ARL_0 = 200, 370$ and 500, respectively. Thus, for example, when 200 reference observations and smoothing constant 0.05 are available and an ARL_0 of 200 is desired, the h for the RE chart is given by 0.02133. From Table 1, we see that for any fixed combination of (m, λ) values, the higher the nominal ARL_0 values, the higher the values of h. Further, for fixed λ, the h increases with the increase in the reference sample size m, and for fixed m, the h also increases with the increase in the λ.

Insert Table 1 about here.

As a Phase II NSPC procedure, it is assumed for the proposed RE chart that the IC process is represented by a Phase I reference sample of size m. The size m would affect how well the IC process is estimated and how well the RE chart performs in Phase II. It should be pointed out that when m is not large, there would be considerable uncertainty in the estimation of the process, which in turn would distort the IC run length distribution of the RE control chart. Figure 1 shows the standard deviation of run length (SDRL) values for the corresponding ARL values in Table 1. We also show the SDRL and ARL for geometric distribution, as the run length distribution can generally be approximated by geometric distribution ([31, 35]). From Figure 1, we can see that the SDRL-ARL profiles would get more similar to that of geometric distribution as m increases. Furthermore, as long as $m \geq 200$, the SDRL values are quite stable for all the λ considered. Therefore, we suggest collecting at least 200 IC observations before Phase II process monitoring.

Insert Figure 1 about here.

To deal with the situation when a sufficiently large reference data set is unavailable, one possible method is the bootstrap method, such as those of [11, 18].

3.3 Sensitivity analysis

For our proposed RE procedure, there are 3 parameters: the change point τ, the size of Phase I observations m and the smoothing parameter λ. To show the effects of these parameters on ARL, the IC distribution is chosen to be one of the following three distributions: $N(0,1)$, $t(3)$ and χ^2_3. Among these distributions, $t(3)$ represents symmetric distributions with heavy tails, and χ^2_3 represents skewed distributions. It is also assumed the pre-specified ARL_0 value is 370. For the three parameters, we choose 2 different levels for each parameter, i.e., $\tau = 10$ or 50, m=100 or 200, λ =1 or 0.1. Figure 2 and Figure 3 show the ARL values under the settings above for mean shifts and variance shifts, respectively.

Insert Figures 2 about here.

Insert Figures 3 about here.

From Figure 2 and Figure 3, comparing the left panel ($\tau = 10$) and the right panel ($\tau = 50$), we can see that the ARL values are generally larger when $\tau = 50$. With similar effect, ARL values are generally larger when $m = 200$. These findings are particularly useful, which implies the OC performance of the proposed RE chart is less affected than IC performance by the number of Phase I observations. Considering λ, it seems that larger λ can obtain better performance.

It seems that the simulation findings above may contradict with some well-known results for EWMA and Shewhart charts. Firstly, it seems that larger λ can obtain better performance, even for some cases of small shifts, which leads to a phenomenon that the Shewhart charts detects small shifts more quickly than the EWMA chart. The possible reasons may be as follows. We can note from Table 1 that the control limits are quite small, even as small as 10^{-2}, which implies that the monitoring statistics RE_n are also quite small. In this case, a smaller value of λ means that the current statistic T_1 receives a smaller weight, and an observation even very far from the IC value may not result in an immediate OC signal. Moreover, as pointed out by [46], the maximum value of the signal resistance, which is defined as the largest standardized deviation (distance) from the IC process value not leading to an immediate OC signal, of an EWMA type chart increases as the value of λ decreases. They also showed through simulation that under a worst-case scenario, a sample mean more than 15 standard errors from the target value does not lead to an immediate OC signal. Therefore, they strongly recommend using EWMA type charts only in conjunction with Shewhart

limits, especially with smaller values of λ, so as to remove much of the adverse effect of inertia.

Secondly, the ARL values are influenced by τ, i.e. the change point, especially, for the Shewhart chart ($\lambda = 1$). It is reasonable that the ARL values are influenced by τ for the EWMA chart ($\lambda < 1$), which is consistent with the literature, such as [52]. For our RE chart with $\lambda = 1$, though it seems like a Shewhart type chart, it is different from the conventional Shewhart chart, which only uses the current observations, because T_1 is still a function of the ranks of all observations up to the current time point N. Moreover, as T_1 is squared difference of the average of the ranks of $\{X_1, X_2, \ldots, X_m\}$ and $\{Y_1, Y_2, \ldots, Y_\tau, Y_{\tau+1}, \ldots, Y_n\}$, the larger τ is, the more of the average of ranks of $\{Y_{\tau+1}, \ldots, Y_n\}$ will be offset by the average of ranks of $\{Y_1, Y_2, \ldots, Y_\tau\}$. In another word, the average of ranks of $\{Y_{\tau+1}, \ldots, Y_n\}$ will be masked to some extent by the average of ranks of $\{Y_1, Y_2, \ldots, Y_\tau\}$. As a consequence, the proposed RE procedure will become more ineffective in reacting to some delayed shifts in the process.

4 Comparison Study

We present some simulation results in this section regarding the performance of our proposed RE procedure and compare it with some competing methods in the literature.

The purpose of this paper is to study a NSPC chart based on individual observation for detecting shifts in both location and scale parameters of a process in Phase II when both of these process parameters are estimated from IC Phase I reference sample. It should be clearly noted that there is no existing meaningful nonparametric EWMA chart for this purpose and comparing the RE procedure with alternative NSPC methods turns out to be difficult due to the lack of an obvious comparable method. This is due to the following two reasons. First, most of the approaches in the literature were designed for the cases where location shifts or variance shifts separately, such as the work of [28, 50, 16, 15, 20, 21]. Second, much of the rank-based work were based on group observations and they can not apply to individual observation case, such as those of [20, 21, 32, 1, 2, 13, 14].

As far as we know, except for NLE procedure of [52], there is not any scheme that is based on individual observation and can monitor the location and the scale parameters of a univariate continuous distribution. Never-

theless, comparing our RE chart to charts that are designed to detect only location changes or only variability changes would be also interesting. When the process distribution is not normal, [3] show that a properly designed EWMA chart is robust to departures from normality, so the robust EWMA (denoted as RoE) is compared with our RE chart. Moreover, [8] propose a precedence chart with runs-type signaling rules and the 2-of-2 version is suggested. By this version of the chart, a signal of shift is delivered when two consecutive medians are both on or above the upper bound or both on or below the lower bound. Although the 2-of-2 version precedence chart is Shewhart type, it is known that the performance of Shewhart type chart with runs rules will be greatly improved. Therefore, we also compare our RE chart with 2-of-2 version precedence chart (denoted as Pre).

Now the proposed RE procedure is compared with NLE procedure of [52], RoE of [3], and Pre of [8]. As the number and variety of OC settings are too large to allow a comprehensive comparison and our goal is to show the effectiveness and sensitivity of the RE chart, and thus we only choose certain representative models for illustration. Following the robustness analysis of [52], we consider the following distributions: the standardized version with mean 0 and standard deviation 1 of one of the following three distributions: $N(0,1)$, $t(3)$ and χ^2_3, as in Section 3.3, which represent two popular symmetric distributions and a well-known asymmetric distribution. The IC Phase I sample size m is chosen to be 20000 and 200, which setting is consistent with [52]. The ARL_0 value is set to 370 and control limits are searched such that the actual ARL_0 approximates 370 well. The zero-state ARL results are shown in Tables 2-4. All the results in this section are obtained from 10,000 Monte-Carlo replications unless indicated otherwise.

Insert Table 2 about here.

Insert Table 3 about here.

Insert Table 4 about here.

From Tables 2-4, in general, the simulation results reveal that for fixed m, λ and a given IC distribution, the OC ARL values decrease sharply with the increasing shift in the location and also with the increasing shift in the scale. This indicates that all these nonparametric charts are reasonably effective in detecting shifts in the location and/or in the scale. However, the performance of the chart (speed of detection) varies depending on the type of shift and the type of IC distribution being considered.

Specifically, the performance of our RE procedure is generally much better than all other charts for most of the location shifts considered, except for

quite large shift size, say $\delta = 3$ for RoE and Pre. For example, when $m = 20000$, $\delta = 0.5$, $\lambda = 0.05$ and the IC distribution is normal, the ARL of proposed RE chart is 14.8, which is about 40% of 36.1 for NLE, 56% of 26.3 for RoE, and 14% of 108 for Pre. When $m = 200$, the ARL reduction of RE chart compared with other charts is also quite large. Although the ARL of RoE is a little smaller than that of RE when $\delta = 3$, RoE is not indeed robust and the actual ARL_0 will deviate from the nominal ARL_0, as shown in [38]. In fact, we adjusted the control limits of RoE such that the actual ARL_0 approximates the nominal ARL_0 well.

As for scale shifts, our RE procedure is generally no better than the compared charts for most of cases, except that it is better than RoE when the IC distribution is χ^2_3. A possible explanation for the inferior position of RE chart may be that we only use the first order of ranks as T_1 in Eq. (2.5), and we may improve the performance of our RE procedure by using second order of ranks, e.g., T_2, which may warrant further research.

Considering the significant ARL reduction for location shifts and if monitoring the location is considered more important than monitoring the variability, our proposed RE chart should be a good alternative. And in practice, a practitioner can not expect whether a process shift is due to location or variability, our RE chart can guard against both location and variability shifts, though not so effective for single scale shifts.

5 Real Application Example

We illustrate the proposed RE procedure using a dataset from an aluminum smelter that produces metallic aluminum from dissolved alumina through a chemical reaction process. The dataset contains the content of SiO_2, Fe_2O_3, MgO, CaO and Al_2O_3 (labeled as x_1, x_2, x_3, x_4 and x_5, respectively) in the cryolite/alumina mixture that goes through the chemical reaction process. More detailed information about this data can be found in [36].

The data have 189 observations. Like many other Phase II SPC procedures, our procedure assumes that observations at different time points are independent of each other. Following the suggestions in [36], we first pre-whiten the data and then try to apply the related procedures considered in the previous section to the pre-whitened data. The Shapiro-Wilk test for checking the normality of the IC data gives p-values of 3.272×10^{-10}, 2.578×10^{-2}, 1.924×10^{-1}, 2.301×10^{-5} and 5.079×10^{-4}, which imply that

the IC data are significantly non-normal for x_1, x_4 and x_5. To this end, the first 95 observations are used as an IC data, and the remaining observations are used for testing.

For our RE procedure, the parameters are chosen such that its ARL_0 equals 200 and it signals a location shift or variability shift in Phase II process monitoring as early as possible. The charting statistics are shown in Figure 4. For the RE chart with $\lambda = 0.1$, the charts give OC signals at the 102nd, 118th, 172nd, 113rd, 102nd observation, while for the RE chart with $\lambda = 1$, the charts give OC signals at the 101st, 118th, 184th, 114th, 101st observation for the 5 contents, respectively. Note that [36] show that the process may go OC around 153. Based on our RE procedure, except for x_3, our charts for other variables give OC signals sooner.

Insert Figure 4 about here.

6 Concluding Remarks

We developed a new NSPC procedure for univariate continuous process in cases when the process distribution cannot be specified beforehand. It integrates a nonparametric test based on works of [24] and [17] with the EWMA model. Compared to some existing control procedures, the proposed procedure is robust to non-normally distributed data, and efficient in detecting various process shifts, especially for location shifts. As it avoids the need for a lengthy data-gathering step and it does not require knowledge of the underlying distribution, our proposed procedure is particularly useful in start-up or short-run situations. Therefore, this is the NSPC procedure that we recommend to use in practice.

It is worth pointing out here that apart from quick detecting abnormal changes, isolating the shifted components or factors that are responsible for the change is also a fundamental task of SPC, especially for a single control chart. For example, in the application example from the previous section, it would be interesting and helpful to determine which factors are responsible for the change of quality, the location or the variability. The problem of isolating the causes of our proposed RE procedure after it triggers a signal warrants further research. Further, although our proposed RE procedure does not require the specification of the IC distribution, it is assumed that an IC dataset is available for estimating the IC parameters or the distribution.

[27, 4] show the need for nonparametric approach to Phase I analysis.

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Appendix

The simulation procedure by which the ARL values (in Sections 3-4) are derived is described in this appendix. The Fortran program used is available from the authors upon request. A user can also code his own computer program by the guidelines below.

Step 0: Input parameters to be specified by the practitioners, including Phase I sample size m, change point position τ, smoothing constant λ and control limit h, which can be obtained by the algorithm in Section 3.2.

Step 1: Generate m IC Phase I sample, $X_1, X_2, \ldots, X_m$.

Step 2: Set $n = 0$, $RE_n = 0$ and $RL = 0$.

Step 3: This step includes 4 different cases as follows.

- If $n \leq \tau$ and $RE_n \geq h$, then go to Step 2.
- If $n \leq \tau$ and $RE_n < h$, then update n to $n = n + 1$, generate one IC Phase II sample Y_n, update RL to $RL = RL + 1$, and calculate RE_n. Go back to Step 3.
- If $n > \tau$ and $RE_n < h$, then update n to $n = n + 1$, generate one OC Phase II sample Y_n, update RL to $RL = RL + 1$, and calculate RE_n. Go back to Step 3.
- If $n > \tau$ and $RE_n \geq h$, then go to Step 4.

Step 4: Set $RL = RL - \tau$.

Step 5: Repeat Step 2-4 B_1 times to obtain the average of the RL, i.e., ARL.

Step 6: Repeat Step 1-5 B_2 times to obtain the average of the ARL values such that the effect of the size m of Phase I sample is negligible.

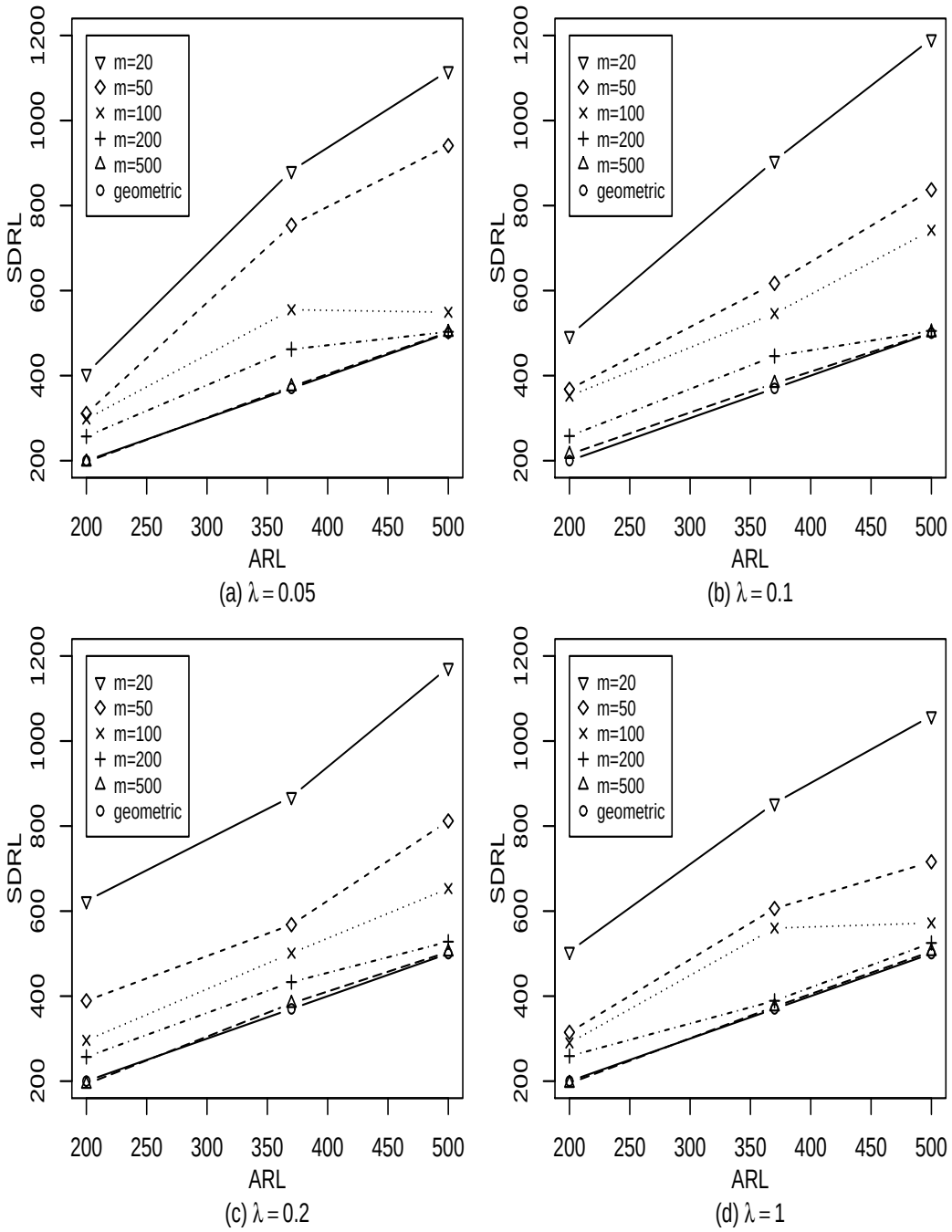

Figure 1: SDRL values versus different ARL values for different m.

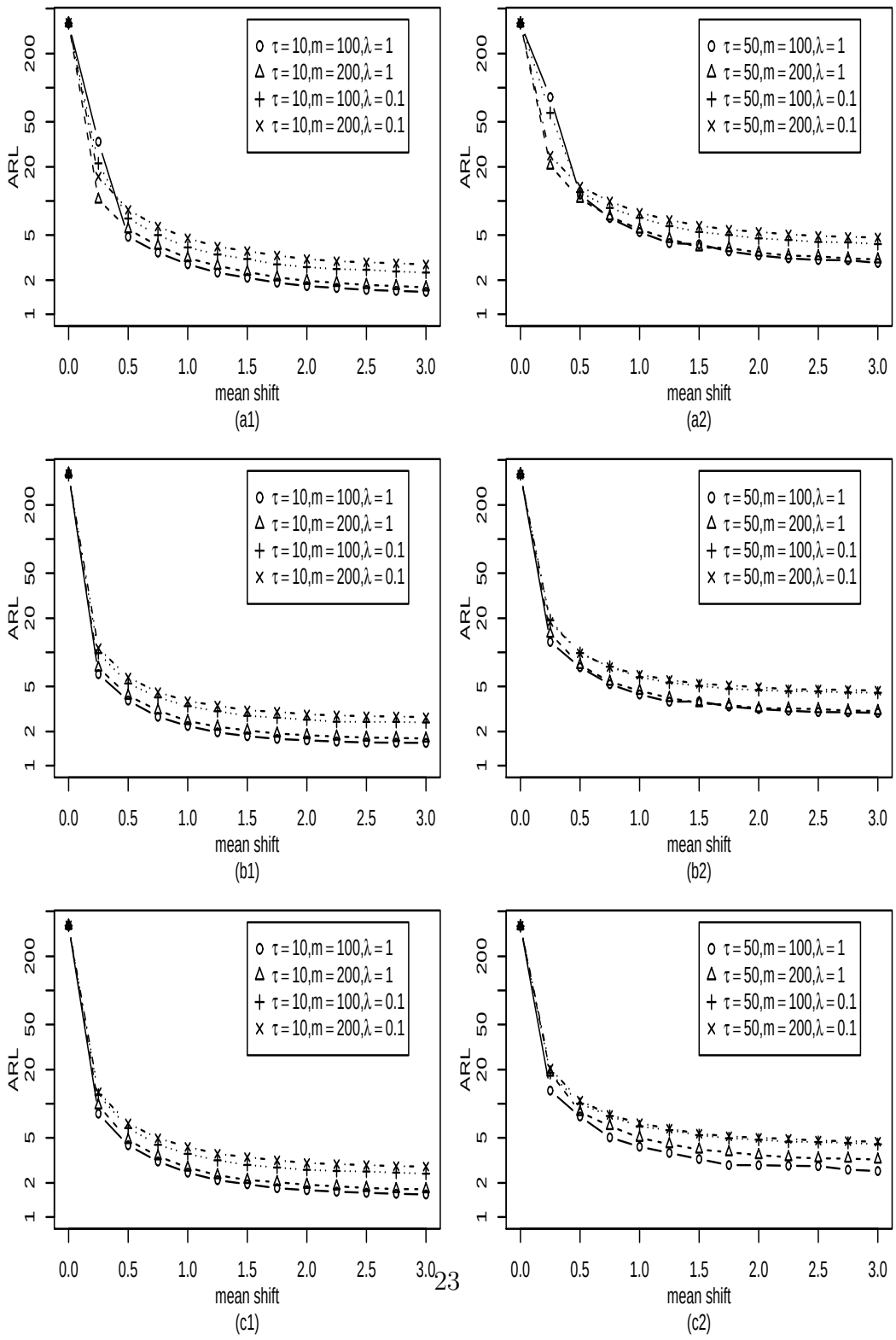

Figure 2: ARL values for different mean shifts when the IC distribution is $N(0,1)$ (a1 and a2), $t(3)$ (b1 and b2) and χ^2_3 (c1 and c2).

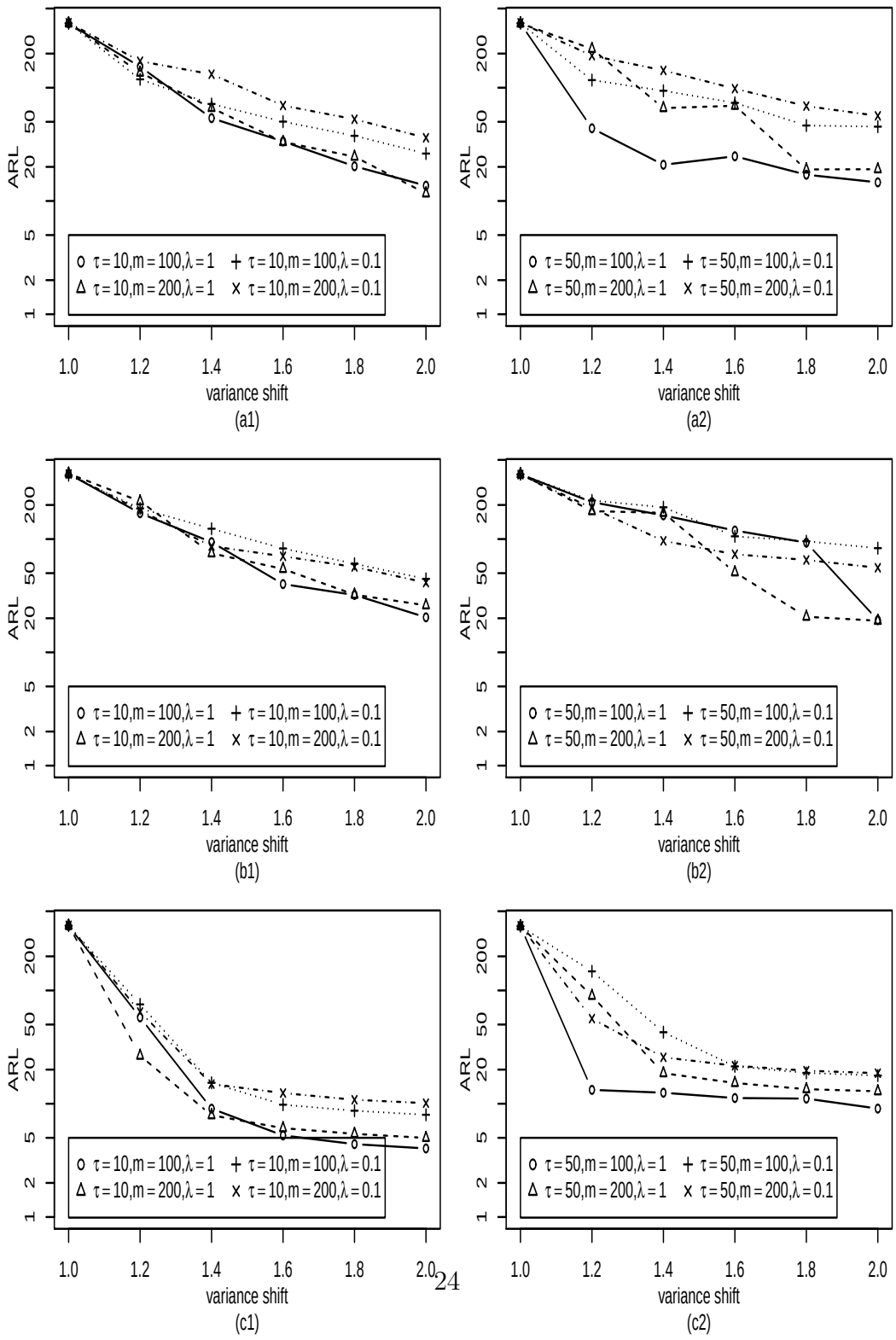

Figure 3: ARL values for different variance shifts when the IC distribution is $N(0,1)$ (a1 and a2), $t(3)$ (b1 and b2) and χ^2_3 (c1 and c2).

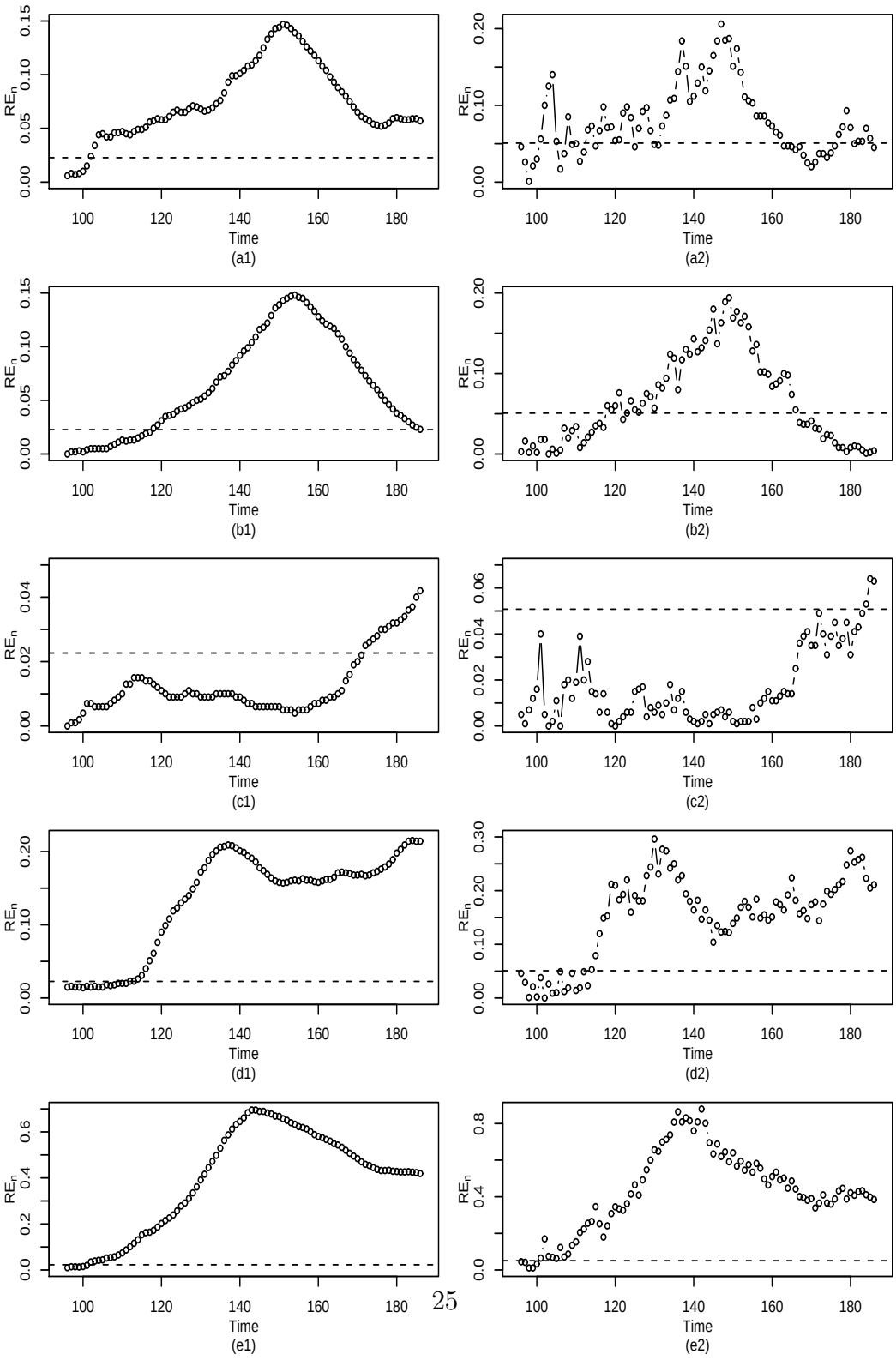

Figure 4: Monitoring statistics of RE procedure with $\lambda = 0.1$ (left column) and $\lambda = 1.0$ (right column) when it is applied to the aluminum data for x_1 (a1 and a2), x_2 (b1 and b2), x_3 (c1 and c2), x_4 (d1 and d2), x_5 (e1 and e2), respectively. In each plot, the horizontal dashed line(s) denotes the control limit(s).

Table 1: The control limits h for various combinations of m, λ and ARL_0.

m	λ	$ARL_0 = 200$	$ARL_0 = 370$	$ARL_0 = 500$
20	0.05	0.00773	0.00926	0.00938
	0.10	0.01131	0.01352	0.01459
	0.20	0.01533	0.01680	0.01977
	1.00	0.03252	0.03430	0.03742
50	0.05	0.01383	0.01424	0.01633
	0.10	0.01617	0.01805	0.01992
	0.20	0.02139	0.02412	0.02619
	1.00	0.04119	0.04854	0.05117
100	0.05	0.01693	0.01834	0.01998
	0.10	0.02104	0.02562	0.02586
	0.20	0.02842	0.03086	0.03350
	1.00	0.05257	0.05664	0.05854
200	0.05	0.02133	0.02338	0.02523
	0.10	0.02664	0.03023	0.03205
	0.20	0.03430	0.03906	0.04023
	1.00	0.05891	0.06650	0.07048
500	0.05	0.02750	0.03258	0.03406
	0.10	0.03516	0.03875	0.04141
	0.20	0.04307	0.04852	0.05045
	1.00	0.06875	0.07779	0.08318

Table 2: The zero-state ARL comparisons among RE, NLE, RoE and Pre under the normal distribution.

		RE		NLE		RoE		Pre
		$N(0,1)$ versus $N(\delta,1)$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
	0.00	370	369	369	370	369	371	371
	0.50	14.8	12.4	36.1	37.7	26.3	28.4	108
20000	1.00	7.07	5.67	14.1	12.2	10.7	9.74	25.7
	1.50	5.02	3.96	7.65	6.54	6.75	5.80	9.10
	2.00	4.24	3.32	4.57	4.01	4.98	4.19	4.59
	3.00	3.49	3.02	2.08	1.95	3.34	2.76	2.39
	0.00	369	370	371	368	372	369	371
	0.50	10.9	9.42	37.2	46.8	31.3	31.1	121
200	1.00	5.22	4.38	14.3	12.5	11.6	9.84	30.7
	1.50	3.72	2.98	9.13	7.86	7.06	5.83	9.82
	2.00	3.23	2.42	6.73	5.79	5.20	4.29	4.81
	3.00	3.01	2.05	4.63	4.07	3.50	2.81	2.40
		$N(0,1)$ versus $N(0,\delta^2)$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
	1.00	370	369	370	370	369	371	371
	1.20	219	199	57.0	58.4	147	126	113
20000	1.40	149	129	23.0	22.4	81.9	63.8	53.3
	1.60	114	90.9	13.6	12.8	53.5	39.5	32.1
	1.80	92.0	73.6	9.47	8.82	38.2	27.6	22.3
	2.00	79.5	63.1	7.12	6.66	29.2	20.6	17.0
	1.00	369	370	370	372	372	369	371
	1.20	191	231	114	115	163	127	106
200	1.40	147	146	41.4	41.2	88.8	67.6	54.3
	1.60	98.9	110	25.8	22.6	59.5	40.0	32.1
	1.80	83.8	81.7	19.4	16.4	42.3	27.9	22.5
	2.00	57.3	61.6	16.1	13.3	32.2	21.2	16.9

Table 3: The zero-state ARL comparisons among RE, NLE, RoE and Pre under the $t(3)$ distribution.

		RE		NLE		RoE		Pre
		$\frac{t(3)}{\sqrt{3}}$ versus $\frac{t(3)}{\sqrt{3}}+\delta$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
	0.00	369	369	372	371	371	371	368
	0.50	10.3	8.43	28.4	26.8	27.5	33.4	148
20000	1.00	5.50	4.35	13.3	10.9	10.6	9.76	28.4
	1.50	4.43	3.42	8.42	6.86	6.63	5.73	6.67
	2.00	4.15	3.16	6.04	4.89	4.87	4.14	3.08
	3.00	3.72	3.04	3.74	3.14	3.28	2.75	2.13
	0.00	371	371	370	371	372	369	371
	0.50	7.73	6.37	25.5	26.2	37.1	50.4	186
200	1.00	4.15	3.28	11.8	10.1	19.4	11.1	42.6
	1.50	3.34	2.45	8.16	7.05	6.95	5.82	7.96
	2.00	3.13	2.19	6.53	5.66	5.15	4.04	3.55
	3.00	3.04	2.05	5.01	4.40	3.64	2.82	2.15
		$\frac{t(3)}{\sqrt{3}}$ versus $\delta \cdot \frac{t(3)}{\sqrt{3}}$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
	1.00	369	369	370	369	371	371	368
	1.20	241	225	127	132	178	144	170
20000	1.40	174	155	56.7	60.3	109	80.2	93.8
	1.60	134	119	34.3	34.4	73.1	55.5	59.6
	1.80	111	96.3	24.2	23.2	62.6	48.6	41.1
	2.00	94.6	76.0	18.8	17.6	43.2	50.1	30.5
	1.00	371	371	370	369	372	369	371
	1.20	281	229	206	207	203	174	179
200	1.40	185	168	95.3	100	191	110	93.9
	1.60	159	117	50.3	52.9	157	83.1	63.6
	1.80	112	101	33.3	32.5	60.1	42.0	41.6
	2.00	98.8	80.2	25.8	23.6	46.2	30.9	30.3

Table 4: The zero-state ARL comparisons among RE, NLE, RoE and Pre under the χ^2_3 distribution.

		RE		NLE		RoE		Pre
		$\frac{\chi^2_3-3}{\sqrt{6}}$ versus $\frac{\chi^2_3-3}{\sqrt{6}}+\delta$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
20000	0.00	371	370	373	370	369	371	373
	0.50	10.8	8.84	30.7	26.8	27.4	29.3	253
	1.00	5.88	4.66	17.9	14.4	11.0	10.2	90.2
	1.50	4.57	3.59	12.3	9.82	6.81	5.96	33.3
	2.00	4.05	3.09	8.91	7.12	4.98	4.22	13.4
	3.00	3.97	2.77	5.11	4.22	3.34	2.77	2.97
200	0.00	371	371	373	366	368	368	370
	0.50	8.06	6.83	22.9	22.9	30.9	29.6	272
	1.00	4.34	3.62	13.9	11.9	11.8	10.4	118
	1.50	3.42	2.76	10.3	8.85	7.37	5.96	40.3
	2.00	3.04	2.35	8.32	7.15	5.15	4.32	15.2
	3.00	3.00	2.00	5.94	5.17	3.44	2.80	3.28
		$\frac{\chi^2_3-3}{\sqrt{6}}$ versus $\delta \cdot \frac{\chi^2_3-3}{\sqrt{6}}$						
		λ						
m	δ	0.05	0.1	0.05	0.1	0.05	0.1	
20000	1.00	371	370	374	372	369	371	373
	1.20	74.8	61.3	11.0	10.9	157	135	51.9
	1.40	43.7	35.0	5.69	5.67	87.6	71.6	23.3
	1.60	33.4	26.5	4.06	4.06	57.1	44.0	15.1
	1.80	28.2	22.4	3.35	3.30	40.5	30.4	11.4
	2.00	25.3	19.8	2.92	2.92	30.9	22.7	9.51
200	1.00	371	371	370	373	368	368	370
	1.20	84.8	102	46.2	49.8	185	145	49.5
	1.40	29.1	23.1	22.9	19.9	99.4	74.3	22.5
	1.60	19.4	15.4	17.0	14.2	62.0	45.5	15.0
	1.80	15.8	12.7	14.1	11.7	44.8	31.2	11.3
	2.00	14.3	11.4	12.4	10.2	34.9	23.6	9.43

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