A comparative study of memory-type control charts based on robust scale estimators

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Abstract

Exponential weighted moving average (EWMA) and cumulative sum (CUSUM) control charts are well known tool for their effectiveness in detecting small and moderate changes in the process parameters. To detect both large and small shifts, a new control structure is often recommended, named as combined Shewhart-CUSUM control chart, which combines the advantages of a Shewhart chart with the CUSUM chart. In this paper, we investigate eleven different standard deviation estimators with the structures of these three types of control charts for monitoring the process dispersion under normal and contaminated normal environments. By applying Monte Carlo simulations, we compare the performance of these memory charts depending on four factors: (1) standard deviation estimator, (2) parent environment, (3) chart type, and (4) change magnitude. Extensive simulations are used to compute and study the run length profiles of these memory charts, including the average, the standard deviation, the several percentiles and the cumulative distribution function curves of the run length distribution. It turns out that there is a significant difference between the run length distribution of the memory chart with estimated parameters and the analogous case with known parameters, even using the adjusted control limits under normal environment, and the difference is more severe when contaminations are present. This difference is gradually diminished when a large number of Phase I samples is used under normality, but it is not true in the contaminated cases.

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1 Introduction

Conventional statistical process control (SPC) charts are usually based on the indispensable assumption of known process parameters. In reality, this assumption is often invalid. There are many real industrial situations in which the process parameters are unknown and are estimated from an in-control (IC) Phase I data. However, the Phase I data can contain unusual observations, which are problematic as they can influence the parameter estimates, resulting in Phase II control charts with less ability to detect changes in the process characteristic. One way to deal with contaminations in Phase I is to use robust estimators, which is not sensitive to contamination when estimating the parameter. In this paper, we concentrate on robust control charts for monitoring the process dispersion when the Phase I data may contain contaminated samples. Many researchers have made contributions to apply robust estimators for monitoring the process dispersion parameter (see Rocke,[1] Tatum,[2] Wu et al.,[3] Riaz and Saghir,[4,5] Schoonhoven et al.,[6] Schoonhoven and Ronald,[7] Nazir et al.,[8] Zwetsloot et al.,[9] Nazir et al.,[10] and Shahriari and Ahmadi[11]).

Control chart is a useful technique which helps in detecting out of control signal in a process and it can either be a memory-type or memory-less control chart. Shewhart-type (memory-less) charts are well known for their effectiveness in detecting large shifts, but are less sensitive to detect small and moderate changes in the process parameters. For this reason, some memory-type control charts are proposed, such as the EWMA control charts and the CUSUM control charts. These charts provide better protection against small and moderate process shifts. To detect both large and small shifts, it is often recommended to combine the Shewhart chart with the CUSUM or EWMA chart. Some of these enhancements may be seen in Lucas,[12] Gibbons,[13] Wu et al.,[14] Capizzi and Masarotto,[15] and Abujiya et al.[16,17] These schemes combine the advantages of a Shewhart chart with the CUSUM or EWMA chart and have wide practical applications. For example, the combined Shewhart-CUSUM control chart is the only statistical procedure that is directly recommended for use in intra-well monitoring by U.S. Environmental Protection Agency (cf. Gibbons[13]). It has been hypothesized that the charts with a desirable property of being sensitive to smaller shifts are more severely impacted by parameter estimation (see Hawkins et al.[18]). Thus, it is worthwhile to make a careful study on the effect of parameter estimation on the memory-type charts' performance as these charts are more efficient than the Shewhart chart for detecting small shifts.

In the SPC literature, it has been demonstrated by several authors that the EWMA and CUSUM control charts have similar performance (e.g., Lucas and Saccucci,[19] Acosta-Mejia et al.,[20] Knoth,[21] and Qiu[22]). However, to the best of our knowledge, the comparisons between the two types of control charts for monitoring the process variance when parameter estimation is subject to contaminations have not been studied so far. Additionally, the combined Shewhart-CUSUM (CS-CUSUM) charts for monitoring the process variability in the case of estimated parameters under contaminated environments have not been considered. The main objective of this research is to compare the Phase II performance of the design structures of EWMA, CUSUM and CS-CUSUM control charts with robust sale estimators under different environments. The effect on the control chart performance is multidimensional since it is not only a result of the accuracy of the parameter estimate but also of the choice of the type of the chart and the size of the shift to be detected. Therefore, our comparative study is based on the following four aspects: 1) The impact of different estimators used to estimate the process standard deviation in Phase I; 2) the impact of different parent environments (normal and contaminated normal) in Phase I; 3) the impact of different types of the control charts designed to monitor the process variability; 4) the impact of different change magnitudes to be detected.

Generally, the average run length (ARL) is the widely used criterion for assessing the performance of control charts. However, because the run length distribution of the memory-type chart with estimated parameters is not geometric, as with the Shewhart chart with known parameters, the ARL should not be used as the sole measure of chart performance. Thus, we present the performance of the chart as measured by the ARL, the standard deviation of the run length (SDRL), and the several percentiles of the run length distribution. Moreover, to get more insight into the run length distribution, we also provide the run-length cumulative distribution function curves. This allows a thorough comparison of the effects of estimating parameters on the performance of the charts studied.

The rest of this paper is organized as follows: In the next section, we give the details regarding the control structure of three memory-type control charts. Section 3 provides a description of the estimators of the standard deviation and different parent environments. In section 4, we derive the Phase II control limits. In section 5, we provide a comprehensive comparison of the three memory-type control charts based on eleven different standard deviation estimators in terms of their run length profiles in uncontaminated and various contaminated cases. After that, we also examine the effect of the Phase I sample size on the performance of the charts studied. Several remarks conclude this paper in Section 6.

2 EWMA, CUSUM and CS-CUSUM charts

Knoth[23] investigated two-sided EWMA dispersion control charts based on R, S^2, S, $\log(S^2)$ and $a+b\log(S^2+c)$, and concluded that the best performance in terms of the average run length profile is given by the S^2 and S EWMA control charts. This is supported also by one-sided results (both EWMA and CUSUM). For application, one should prefer S^2 and S. In this paper, we therefore consider the design structures of the EWMA, CUSUM and CS-CUSUM charts based on S for monitoring the process variability, referred to as the EWMA-S, CUSUM-S and CS-CUSUM-S charts, respectively. Note that when monitoring the process dispersion, it is generally of interest in detecting an increase in the process variance, as these situations indicate process degradation. Thus, in this paper we confine ourselves to this case. The current section contains the details about these three one-sided memory structures.

Let $\boldsymbol{Y_t} = (Y_{t1}, Y_{t2}, \cdots, Y_{tn})$ be a Phase II sample of size n at time t from a process where each observation follows a normal distribution $N(\mu, (\delta\sigma)^2)$. Without loss of generality, we use $\mu = 0$ and $\sigma = 1$ throughout this article. δ is the magnitude of the process standard deviation shift. When $\delta = 1$, the standard deviation of the process is IC; otherwise, a shift in process standard deviation occurs (out of control situation). To make these control chart procedures independent of the particular value of σ, the chart statistics are expressed in standardized units. Let $\bar{Y}_t = \sum_{j=1}^{n} Y_{tj}/n$ and $S_t = (\sum_{j=1}^{n}(Y_{tj} - \bar{Y}_t)^2/(n-1))^{1/2}$ be the tth sample mean and sample standard deviation.

2.1 Design of EWMA-S control charts

To detect upward dispersion shifts, the charting statistic of the upper-sided EWMA-S chart with the re-starting mechanism can be defined by

$$E_t = max[(1-\lambda)E_{t-1} + \lambda(S_t/\sigma), c_4(n)]. \tag{1}$$

We set $E_0 = E(S_t/\sigma) = c_4(n)$, where n is the sample size and $c_4(n)$, the bias correcting coefficient, is defined by

$$c_4(n) = \sqrt{\frac{2}{n-1}}\frac{\Gamma(n/2)}{\Gamma((n-1)/2)}. \tag{2}$$

The upper asymptotic control limit of the EWMA-S chart is computed as

$$h_e = c_4(n) + L\sqrt{1-c_4^2(n)}\sqrt{\frac{\lambda}{2-\lambda}}, \tag{3}$$

where the factor L controls the width of the control limit, and λ is the smoothing parameter. An OC is signaled by the EWMA-S chart when E_t plots beyond the limit h_e.

2.2 Design of CUSUM-S control charts

The upper one-sided CUSUM-S statistic is defined as

$$Z_t = max[0, Z_{t-1} + (S_t/\sigma) - k_c], \tag{4}$$

and the starting value of the CUSUM-S is given by $Z_0 = 0$. Here, k_c is the reference value. Based on Tuprah and Ncube,[24] the value of k_c is generally taken to be half between the expected values of S_t/σ given σ and $\delta\sigma$, where δ is the size of the anticipated process dispersion shift that needs to be detected quickly. Accordingly, $k_c = c_4(n)(1+\delta)/2$. The process is deemed to be OC and it is concluded that the process dispersion has increased if $Z_t > h_c$, where h_c determines the decision interval.

2.3 Design of CS-CUSUM-S control charts

The CS-CUSUM-S chart is a mixture of a Shewhart-S chart and CUSUM-S chart. An upper one-sided CS-CUSUM-S chart gives an out of control (OC) signal when either

$$(S_t/\sigma) > UCL, \tag{5}$$

or

$$Z_t = max[0, Z_{t-1} + (S_t/\sigma) - k_c] > h_{cs}, \tag{6}$$

with $Z_0 = 0$ as the starting value of the CUSUM statistic. UCL is the Shewhart upper control limit, h_{cs} is the control limit of the upper CUSUM chart, and k_c is also computed as $k_c = c_4(n)(1+\delta)/2$.

3 Phase I estimators of dispersion

In practice, The value of σ is usually unknown and has to be estimated from samples taken when the process is assumed to be in control. This stage in the control-charting process is called Phase I. Let $\hat{\sigma}$ be an unbiased estimator of the process dispersion parameter σ based on k Phase I samples of size n, which are denoted by

$\boldsymbol{X_t} = (X_{t1}, X_{t2}, \cdots, X_{tn}), t = 1, 2, \cdots, k$. We set $k = 50$ and $n = 5$ for illustration in this paper. There are many choices of $\hat{\sigma}$. Schoonhoven et al.[6] studied various standard deviation estimators for designing the control chart and provided a comprehensive analysis on their efficiency under both uncontaminated and contaminated environments for different phases. Schoonhoven and Ronald[7] proposed several adaptive trimmers which perform substantially better than the traditional estimator and some robust proposals when contaminations are present. In deriving the estimators of the process standard deviation parameter, we will consider some of the estimators discussed in Schoonhoven et al.[6] as well as some other robust estimators that are variants of Schoonhoven and Ronald.[7] In the current section, the details about the estimators used in this study are given. The pooled sample standard deviation is the most widely used dispersion statistic for control charts and is defined as

$$S_p = \sqrt{\frac{1}{k}\sum_{t=1}^{k} S_t^2}. \tag{7}$$

The unbiased estimator is given by $S_p/c_4(k(n-1)+1)$. Mahmoud et al.[25] showed that, in the multiple samples case, this estimator is the most efficient unbiased estimator when data are normally distributed.

Another commonly used dispersion statistic is the mean of the sample standard deviations and is defined as

$$\overline{S} = \frac{1}{k}\sum_{t=1}^{k} S_t. \tag{8}$$

The unbiased estimator of σ is given by $\overline{S}/c_4(n)$.

We also take into account an estimator proposed by Rousseeuw and Croux,[26] which can be written as

$$Q_{nt} = 2.2219 * \{|X_{ti} - X_{tj}|; i < j\}_{(p)} \text{ where } p = \binom{[n/2]+1}{2}. \tag{9}$$

Here, the symbol $[x]$ represents the 'floor' function and is defined as the largest integer less than or equal to x. Q_{nt} is 2.2219 times the pth order statistic of the n-choose-2 distances between data points of sample t. The reason of choosing Q_{nt} is that it has the highest possible breakdown point (50%) and its efficiency under normality is very high (about 82%). We use the mean of the sample Q_{nt}'s, given by

$$\overline{Q_n} = \frac{1}{k}\sum_{t=1}^{k} Q_{nt}. \tag{10}$$

An unbiased estimator of σ is given by $d_n\overline{Q_n}$, the correction factor d_n is obtained by simulation. Extensive tables of d_n can be found in Croux and Rousseeuw.[27]

The next dispersion estimator is based on the mean of the sample average absolute deviation from the median, given by

$$\overline{ADM} = \frac{1}{k}\sum_{t=1}^{k} ADM_t, \tag{11}$$

where ADM_t is the average absolute deviation to the median of sample t, given by

$$ADM_t = \frac{1}{n}\sum_{j=1}^{n} |X_{tj} - M_t|, \tag{12}$$

with M_t the median of sample t. Based on Wu et al.,[3] Riaz and Saghir[5] and Schoonhoven et al.,[6] the ADM chart is less sensitive to non-normality than either the R or S chart, and it provides quite a satisfactory performance when there are no or only a few contaminations in the Phase I normal data. An unbiased estimator of σ is given by $\overline{ADM}/t_2(n)$. Wu et al.[3] had provided a proposition for the analytical result of $t_2(n)$ in the form of an integral.

Gini[28] proposed the following estimator of σ:

$$G_t = \sum_{j=1}^{n-1}\sum_{l=j+1}^{n} |X_{tj} - X_{tl}|/(n(n-1)/2), \tag{13}$$

which is the mean absolute difference between any two observations in the sample t, known as Gini's mean difference. for the multiple-sample case, we use the average of the sample differences (G_t's), given by

$$\overline{G} = \frac{1}{k}\sum_{t=1}^{k} G_t. \tag{14}$$

Since $E(G_t) = \frac{2\sigma}{\sqrt{\pi}}$, so $\frac{\sqrt{\pi}}{2}\overline{G}$ is the unbiased estimator of σ.

Next, we evaluate a robust estimator based on the sample interquartile ranges, which are defined as

$$IQR_t = X_{t(b)} - X_{t(a)}, \tag{15}$$

where $X_{t(o)}$ denotes the oth ordered value in sample t, $a = [n/4] + 1$, $b = n - a + 1$. This definition is following Rocke[1] and slightly differs from the usual definitions for the IQR. We consider the mean of the sample IQR's, given by

$$\overline{IQR} = \frac{1}{k}\sum_{t=1}^{k} IQR_t, \tag{16}$$

and for $n = 5$, an unbiased estimator of σ is given by dividing $\overline{IQR}$ by 0.990.

We also consider a version of the 20% trimmed mean of the sample IQR's as proposed by Rocke[1] and Zwetsloot et al.,[9] which can be written as

$$\overline{IQR}_{20} = \frac{1}{k-2[k\alpha]}\sum_{o=[k\alpha]+1}^{k-[k\alpha]} IQR_{(o)},\ \ \alpha = 20\%, \tag{17}$$

where $IQR_{(o)}$ denotes the oth ordered value of the sample IQR's. The normalizing constant is 0.926 for $n = 5$. This estimator reduces the effect of outlying subgroups and outliers in subgroups, although it is not very efficient under normality.

Tatum[2] proposed a robust estimator, which is based on a variant of the biweight A estimator. Many researchers studied the robustness of this estimator (cf. Schoonhoven et al.,[6] Schoonhoven and Ronald[7] and Zwetsloot et al.[9]) and concluded that the chart based on this estimator has a comparable performance in both normal and contaminated normal cases, although this estimator is relatively complicated in its use. This method begins by centering each observation on its sample median M_t, calculating the residuals in each sample: $res_{tj} = X_{tj} - M_t$. Apparently, if n is odd, each sample contains one residual equal to zero, which is dropped. The total $n'k$ residuals, with $n' = n-1$ when n is odd and $n' = n$ when n is even, are weighted by $u_{tj} = \frac{h_t res_{tj}}{cM^*}$, where M^* is the median of the absolute values of the $n'k$ residuals, and

$$h_t = \begin{cases} 1 & E_t \leq 4.5, \\ E_t - 3.5 & 4.5 < E_t \leq 7.5, \\ c & E_t > 7.5, \end{cases} \tag{18}$$

$E_t = IQR_t/M^*$, and c is a tuning constant. Tatum's estimator is given by

$$S_c^* = \frac{n'k}{\sqrt{n'k-1}}\frac{\sqrt{\sum_{t=1}^{k}\sum_{j:|u_{tj}|<1} res_{tj}^2(1-u_{tj}^2)^4}}{|\sum_{t=1}^{k}\sum_{j:|u_{tj}|<1}(1-u_{tj}^2)(1-5u_{tj}^2)|}. \tag{19}$$

We choose $c = 7$ in this paper, since the value is shown to be robust against various contaminations by Tatum.[2] An unbiased estimator of σ is given by $S_c^*/d(n,k,c)$, where $d(5,50,7) = 1.068$. The resulting estimator is denoted by $D7$ as in Tatum.[2]

To obtain a robust estimator of σ, many researchers recommended the use of a two-step procedure (i.e. screening method), namely using a robust estimator to estimate an initial dispersion and compute the control limits, deleting any subsample that exceeds the control limits, and then an efficient estimator for post-screening estimation. Rocke[1] showed that "examination of the data and recomputation of control limits adds to the effectiveness of any of the procedures that one may choose." In this paper, we therefore propose three screening methods, which are some variants of Schoonhoven and Ronald,[7] but we use an efficient unbiased estimator, the mean of the sample standard deviations, based on the screened data as the final estimator of σ. The first is a procedure for subgroup screening, which consists of the following steps:

1. Choose $\overline{ADM}/t_2(n)$ to be an initial unbiased estimator of σ, because $\overline{ADM}$ has very similar efficiency as the traditional estimator S_p under normality, and it is more robust against outliers. This estimator is then used to construct a Phase I standard deviation control chart so that the subgroups can be screened.

2. Adopt $S/c_4(n)$ as the charting statistic, together with the control limits $\widehat{UCL} = U_n\overline{ADM}/t_2(n)$ and $\widehat{LCL} = L_n\overline{ADM}/t_2(n)$. We derive the factors U_n and L_n from the 0.99865 and 0.00135 quantiles of the distribution of $S/c_4(n)$ through 100,000 Monte Carlo simulations. The factors for the limits are $U_n = 2.2406$ and $L_n = 0.1735$ for $n = 5$. Any subgroup for which the corresponding statistic $S/c_4(n)$ exceeds the Phase I control limits is deleted and $\overline{ADM}$ is recomputed from the remaining subgroups. We continue until all subsample standard deviations fall within the limits.

3. Obtain the final estimator of σ from the screened data. We select an efficient estimator, based on the remaining samples, that is

$$\overline{S}_{sub} = \frac{1}{k'} \sum_{t \in K} S_t/c_4(n), \tag{20}$$

   with K the set of samples that are not excluded, k' is the number of nonexcluded samples, and S_t is the standard deviation of sample t. To obtain an overall unbiased estimator of σ from the screened data, the final normalizing constant is 0.999 for $n = 5$ (obtained through 100,000 Monte Carlo simulations). The resulting estimator is denoted by $\overline{ADM}s$.

Since the above procedure trims off samples instead of individual observations, it performs better when there are localized disturbances. For diffuse outliers, Schoonhoven and Ronald[7] proposed an individuals chart that is expected to detect outliers more quickly. Accordingly, the second procedure is a variant of Schoonhoven and Ronald.[7] The algorithm consists of the following steps:

1. To measure the variability within and not between subgroups, this method begins by determining the residuals in each sample by subtracting the subgroup median from each observation in the corresponding subgroup. Then an individuals chart of the residuals is constructed with the control limits:

$$\widehat{UCL} = 3\overline{ADM}/t_2(n),\ \widehat{LCL} = -3\overline{ADM}/t_2(n), \tag{21}$$

   where $\overline{ADM}/t_2(n)$ is used to estimate σ, and for simplicity, the factors for the individuals chart are 3 and -3. The residuals that fall outside the control limits are excluded from the dataset. Then the median values of the adjusted subgroups are determined, the residuals and the control limits are recomputed using the remaining observations. This repeats until all residuals fall within the control limits.

2. Use an efficient estimator of σ as the post-screening estimation, that is

$$\overline{S}_{ind} = \frac{1}{k'} \sum_{t \in K} S'_t / c_4(n'_t), \tag{22}$$

   with K the set of samples that are not excluded (In rare occasion, all observations in a sample are deleted, so the corresponding sample is excluded), k' is the number of nonexcluded samples, n'_t is the number of nonexcluded observations in sample t, and S'_t is the standard deviation of the remaining n'_t observations. The normalizing constant necessary to obtain an overall unbiased estimator of σ is 0.976 for $n = 5$. The resulting estimator is denoted by $\overline{ADM}i$.

The last procedure considered is an algorithm that combines the use of an individuals chart with subgroup screening. Our approach follows a similar procedure as in the work of Schoonhoven and Ronald.[7] The procedure involves the following steps for an efficient implementation:

1. Similar to that of $\overline{ADM}s$ in step 1 (i.e., choose $\overline{ADM}/t_2(n)$ to be an initial unbiased estimator of σ for subgroup screening).

2. Adopt the standard deviation after trimming the observations in each sample as a charting statistic, which is defined as

$$S_\alpha = \Big(\frac{1}{n - 2[n\alpha] - 1} \sum_{o=[n\alpha]+1}^{n-[n\alpha]} (X_{t(o)} - \bar{X}'_t)\Big)^{1/2}, \tag{23}$$

where

$$\bar{X}'_t = \frac{1}{n-2[n\alpha]} \sum_{o=[n\alpha]+1}^{n-[n\alpha]} X_{t(o)}, \tag{24}$$

with $X_{t(o)}$ the oth ordered value in sample t, and α is the trimming percentage. In this study, we take $\alpha = 20\%$. We employ S_{20} for screening purposes instead of IQR used in Schoonhoven and Ronald,[7] because S_{20} is expected to retain more original sample information from the data than IQR. The constant required to obtain an unbiased estimator of σ based on S_{20} is 0.520 for $n = 5$. Plot the $S_{20}/0.520$s of the Phase I samples on the standard deviation control chart with control limits $\widehat{UCL} = U_n\overline{ADM}/t_2(n)$ and $\widehat{LCL} = L_n\overline{ADM}/t_2(n)$. The factors U_n and L_n derived from the 0.99865 and 0.00135 quantiles of the distribution of $S_{20}/0.520$ are 3.2169 and 0.0349, respectively. The subgroup screening is continued until all $S_{20}/0.520$s fall within the limits. Simulation revealed that the resulting estimator of σ is slightly biased and the normalizing constant is 0.999 for $n = 5$.

3. Use the above resulting estimator to screen observations with an individuals chart, which is the similar procedure used to derive $\overline{ADMi}$ (step 1).

4. Use an efficient estimator of σ as the post-screening estimation, which is the similar procedure used to derive $\overline{ADMi}$ (step 2). The normalizing constant necessary to obtain an overall unbiased estimator of σ is 0.976 for $n = 5$. The resulting estimator is denoted by $\overline{ADMsi}$.

In practice, when the data are used to estimate the process parameters, they may be contaminated, such as outliers, step changes or other contaminations. Many different contaminations are studied in the literature (cf. Tatum,[2] Schoonhoven et al.,[6] Schoonhoven and Ronald,[7] Chen and Elsayed[29] and Zwetsloot et al.[9]). In this paper, we consider the uncontaminated case (i.e., the entire Phase I data are from the $N(0,1)$ distribution) and six types of disturbances, including three diffuse scenarios and three localized scenarios, which are given as follows:

1. Diffuse symmetric variance disturbances: each observation has a 95% probability of being drawn from the $N(0,1)$ distribution and a 5% probability of being drawn from the $N(0,2.5^2)$ distribution.

2. Diffuse asymmetric variance disturbances: each observation is drawn from the $N(0,1)$ distribution and has a 5% probability of having a multiple of a χ^2_1 variable added to it, with the multiplier equal to 1.5.

3. Diffuse mean disturbances: each observation has a 95% probability of being drawn from the $N(0,1)$ distribution and a 5% probability of being drawn from the $N(2.5,1)$ distribution.

4. Localized variance disturbances: all observations in a sample have a 95% probability of being drawn from the $N(0,1)$ distribution and a 5% probability of being drawn from the $N(0,2.5^2)$ distribution.

5. A single step shift in the variance: all observations in the last three Phase I samples are drawn from the $N(0,2.5^2)$ distribution.

6. Multiple step shifts in the variance: at each time point, the sample has a 1.8% probability of being the first three consecutive samples drawn from the $N(0,2.5^2)$ distribution. After any such step shift, each sample again has a 1.8% probability of being the start of another step shift. If a shift occurs at the end of the Phase I data set, for example, at $k-1$, only samples $k-1$ and k will be contaminated. This model is adopted from Zwetsloot et al.[9] for the determination of the probability of 1.8% such that all six contaminated scenarios have an approximate contamination rate of 5%.

4 Design and derivation of Phase II control limits of the charts

The above Phase I estimators are used to design the Phase II control charts, say EWMA-S, CUSUM-S and CS-CUSUM-S charts. The IC standard deviation parameter σ in the three charts (i.e. Equations (1), (4), (5) and (6)) is unknown and must be estimated by $\hat{\sigma}$, which is any estimator from section 3 of the process dispersion parameter. To ensure a fair comparison, using a subgroup size of $n = 5$ and the IC ARL (ARL_0) of 370, all the control charts are designed to detect 20% increases in process standard deviation. We derive the optimal parameters for each chart from σ known case. Specifically, for the EWMA-S chart, for a given ARL_0 of 370, and $\lambda = 0.05(0.005)1$, the optimal (λ, L) combination would minimize the OC ARL (ARL_1) with respect to the specific shift $\delta = 1.2$, subject to the chosen ARL_0 constraint. The optimal smoothing parameter value is $\lambda = 0.08$ through simulation. The reference value for the CUSUM-S and CS-CUSUM-S charts is obtained directly from $k = c_4(n)(1+\delta)/2 = 1.034$ for the particular shift $\delta = 1.2$.

Several recent studies have shown that the number of Phase I samples required for a Phase II control chart with estimated parameters to perform properly may be prohibitively high. Another more practical way suggested by many authors, e.g. Mahmoud and Maravelakis[30] and Jones,[31] is to adjust the control limits of the chart so that it produces the desired ARL_0 when only a small number of samples are available. In order to have a fair comparison, the control limits are adjusted for each of the eleven estimators to produce an ARL_0 of 370 under uncontaminated normal environment. We simulate the factors L for the EWMA-S, h_c for the CUSUM-S and UCL and h_{cs} for the CS-CUSUM-S control chart, corresponding to each standard deviation estimator using $k = 50$ Phase I samples of sizes $n = 5$ from an uncontaminated normal environment. Values of these factors are given in Table 1. Note that, for the CS-CUSUM-S chart, we set $UCL = 2.10$, taking inspiration from Abujiya et al.,[17] and for simplicity we only adjust the value of h_{cs} (the control limit of the CUSUM component) to achieve $\text{ARL}_0 = 370$ for any estimator $\hat{\sigma}$. When it comes to contaminated situations, we keep the control limits the same as we have used for the corresponding uncontaminated cases.

[Insert Table 1 about here]

5 Performance comparisons

In this section, EWMA-S, CUSUM-S and CS-CUSUM-S charts corresponding to the estimated parameters cases are presented along with the effects of estimation error on their performance. The most commonly used metrics of the performance of the control charts are the ARL and SDRL. However, since the run length distribution of the charts with estimated parameters is usually highly right skewed, we also study the percentiles of the run length distribution, including the 10th, 50th and 90th percentiles, to obtain more insights on the charts' performance.

5.1 Run length behaviour

Let RL be the random variable denoting the run length of the control chart. For a given Phase I dataset of k samples of size n, we determine the estimator of the standard deviation $\hat{\sigma}$, and the conditional run length distribution can be written as $Pr(RL = rl|\hat{\sigma})$, where rl is a realization of RL. In order to evaluate the overall run length behavior of the control chart, we study the unconditional run length distribution, which can be obtained by integrating over the distribution of $\hat{\sigma}$

$$Pr(RL = rl) = \int_0^\infty Pr(RL = rl|\hat{\sigma})f(\hat{\sigma})d\hat{\sigma}, \tag{25}$$

where $f(\hat{\sigma})$ is probability density function of $\hat{\sigma}$. It is not easy to get analytical results for such a formulation. Therefore, we employ Monte Carlo simulations to approximate the unconditional run length distribution of the proposed memory-type control charts with estimated parameters.

We consider small to large upward shifts in the standard deviation $\delta\sigma$ in Phase II, namely, δ equal to 1.1, 1.2, 1.4 and 1.8. For the Phase I uncontaminated environment and six contamination scenarios presented in section 3, the IC and OC performance characteristics of the run length distribution, including the ARL, the SDRL and several percentiles, are obtained on the basis of 100,000 replicates. The simulation results are shown in Tables 2-8. Further, to get more visual insights into the IC and OC run length distributions, Figures 1-7 plot the IC and OC run-length cumulative distribution function (CDF) curves of the EWMA-S chart with both known and estimated parameters for the uncontaminated and six contaminated cases, respectively. In each figure, the first plot depicts the CDF of IC run length distribution for $RL \leq 1000$, and the other two plots show the OC CDF curves, respectively for $\delta = 1.1$ and 1.4 when $RL \leq 200$ and $RL \leq 30$, which corresponding to early detection. Note that, the CUSUM-S and CS-CUSUM-S charts are not included in these figures, because the general patterns for their CDF curves are similar to that of the EWMA-S chart and thus, only the results of the EWMA-S chart are provided for illustration. It is also worth mentioning that in Table 2 and Figures 1-7, we also provide the simulation results when the IC σ is known (entitled σ in Table 2 and Figures 1-7), which can be regarded as a basis for comparison. We judge the performance of the proposed charts as good, provided that their run length distribution profiles are closer to that of σ known case.

[Insert Tables 2-8 about here]

[Insert Figures 1-7 about here]

5.2 Simulation results comparisons

We compare the performance of the aforementioned control charts based on standard deviation estimator, parent environment, chart type and change magnitude. The comparisons that cover the findings of Tables 2-8 and Figures 1-7 are summarized as follows.

1. Impact of different standard deviation estimators.

   First, we compare the differences between the estimators in the situation where the Phase I data are uncontaminated. From Table 2 and Figure 1, it is observed that the SDRL values for all charts with estimated parameters are larger than those with known parameters, and the actual run length distribution for the estimated-parameter case behaves differently from the known-parameter case. For example, by adjusting the control limits, $\text{ARL}_0 = 370$ is achieved for all charts under normality, but the 10th and 50th percentiles of the IC run length distribution of the chart with estimated parameters are smaller than those with known parameters. This indicates that, in terms of the percentiles, the false alarm rates of the chart with estimated parameters are higher than the analogous known-parameter case though they have the same ARL_0 value. In other words, the ARL is not a typical index of the run length for an IC process. Similar findings still hold for small shifts.

When the Phase I data are uncontaminated, among the estimators under consideration, the estimators S_p, $\overline{S}$, $\overline{ADM}$, $\overline{G}$, $D7$, $\overline{ADMs}$, $\overline{ADMi}$ and $\overline{ADMsi}$ perform relatively well, whereas the charts based on the estimators $\overline{Q_n}$, $\overline{IQR}$ and $\overline{IQR}_{20}$ have the worse performance. The SDRL values of the latter charts are larger than the formers, and these charts achieve the special ARL_0 with elevated probabilities of very short and very long runs. For example, the 10th percentile of the IC run length distribution is 8 in the EWMA-S chart based on the estimator $\overline{IQR}_{20}$ compared with 19 in the EWMA-S chart when the estimator is S_p. Note that, from Figure 1, one can see that under the OC model, the CDF curves of the EWMA-S charts based on the estimators $\overline{Q_n}$, $\overline{IQR}$ and $\overline{IQR}_{20}$ are higher than the other estimators in short-runs, however, this "advantage" is mainly due to very large short-run false alarms due to randomness.

The performance of the charts in the case of contaminated data are tabulated in Tables 3-8 and Figures 2-7. The results provide surprising insights into the significant deterioration of the charts' performance due to the contamination, that is, a substantially increased ARL_0, ARL_1 and SDRL for all charts with estimated parameters. The reason is that σ is much more overestimated when the Phase I data are contaminated. As a result, the upper control limit will become higher and hence its sensitivity to find an increase in the standard deviation in Phase II reduces. As it is obvious in the figures, the shape of the run length distribution for the estimated-parameter case changes more severely from the shape for the known-parameter case under the contaminated environment as compared to that under the uncontaminated environment, where the severity is more pronounced when the process is IC or slightly OC. The main points of the impact of different estimators in the contaminated cases are summarized as follows.

- The performance of the traditional chart with S_p estimator is the most seriously affected by the presence of contamination. For example, the ARL_0 of the EWMA-S, CUSUM-S and CS-CUSUM-S charts based on the estimator S_p are seen to be nearly 12.04, 11.88 and 10.49 times the target ARL_0 of 370 respectively, when diffuse symmetric variance disturbances are present in Phase I.

- Among the robust estimators, the ARL_1 values of the charts based on the estimator $\overline{IQR}_{20}$ are relatively small in all contaminated cases considered, but when the process is IC, the false alarm rates of these charts in short-runs are rather high compared with the known-parameter case and the charts based on the other estimators. Moreover, the charts based on $\overline{IQR}_{20}$ perform the worst when there are no contaminations. Based on these reasons, we do not recommend using $\overline{IQR}_{20}$.
- The charts based on the estimators $\overline{ADM}$ and $\overline{G}$ perform equally well as the traditional charts in the uncontaminated case, but not very well in contaminated situations.
- The charts based on the estimator $D7$ are efficient under normality and perform relatively well in most contaminated scenarios.
- The charts based on the estimators supplemented with a screening method, i.e. the estimators $\overline{ADMs}$, $\overline{ADMi}$ and $\overline{ADMsi}$ considered in this paper, perform well under normality and outperform any of the other charts in most contaminated situations. When diffuse disturbances are likely to occur in Phase I, the charts based on $\overline{ADMi}$ perform better. On the other hand, the charts based on $\overline{ADMs}$ have the better performance when there are localized disturbances. Moreover, from Figures 1-7, it is observed that the CDF curves of the EWMA-S charts with the estimators $\overline{ADMsi}$ and $\overline{ADMi}$ are not distinguishable in diffuse contaminated situations, and under localized contaminated environments, the CDF curves of the EWMA-S charts based on $\overline{ADMsi}$ are similar to those based on $\overline{ADMs}$. It reveals that the estimator $\overline{ADMsi}$, which combines the advantages of the above two estimators, is robust against both types of disturbances.

In summary, when the Phase I data are uncontaminated, the charts with all estimators show similar performance except that the charts based on $\overline{Q_n}$, $\overline{IQR}$ and $\overline{IQR}_{20}$ have the worse performance under normality. When there are diffuse contaminations, the best OC behavior is obtained with $\overline{ADMi}$ as estimator. When there are localized contaminations, the charts based on the estimator $\overline{ADMs}$ have the best performance. If it is unknown what type of contaminations are present in Phase I, the proposed estimator $\overline{ADMsi}$ is considered to be the best choice for a Phase I analysis, since it is robust against both types of contaminations and the charts based on this estimator provide quite a robust overall performance.

2. Impact of different parent environments.

   In practice, when estimating the unknown process parameters in Phase I, one may encounter with different types of parent environments. In this paper, we consider the uncontaminated normal environment and six contaminated normal environments as described in section 3. We distinguish between diffuse and localized special causes of variation. We evaluate three diffuse scenarios and three localized scenarios (i.e., one localized variance disturbance and two sustained shift scenarios). A general conclusion of the impact of different parent environments is summarized as follows.

   When diffuse mean disturbances or multiple step shifts in the variance is present in Phase I, the most of the charts perform worse than those when the other contaminations are present in Phase I. It indicates that a much larger value of σ estimated in Phase I results in a higher value for the upper control limit and hence the ability of detecting an increase in the standard deviation under the two scenarios reduces a lot.

3. Impact of different memory-type control charts.

   The performance of the design structures of the EWMA-S, CUSUM-S and CS-CUSUM-S charts with estimated parameters under different environments are comprehensively compared. A general conclusion of the impact of three Phase II memory-type control charts is summarized as follows.

   - When the Phase I data are uncontaminated, the performance of the EWMA-S and CUSUM-S charts with both known and estimated paremeters are comparable, and slightly better than the CS-CUSUM-S chart for detecting small shifts in the process variability ($\delta \leq 1.2$), however, for detecting relative large shifts in variance ($\delta \geq 1.4$), the converse is true. When the process is IC, the CS-CUSUM-S chart provide the best IC run length behavior, followed by the CUSUM-S chart.
   - When the Phase I data are contaminated, from Tables 3-8, it is observed that the IC performance of the EWMA-S, CUSUM-S and CS-CUSUM-S charts with estimated parameters are highly affected by the presence of the contaminations. For all three charts with estimated parameters, the ARL_0 values are much higher than the nominal ARL_0 of 370, and the IC SDRL

($SDRL_0$) values are also dramatically increased. The ARL_0 values of the CS-CUSUM-S charts are relatively less affected as compared to the ARL_0 values of the EWMA-S and CUSUM-S charts. The CUSUM-S chart performs slightly better than the EWMA-S chart when the process is IC in most cases.

- When the Phase I data are contaminated, the EWMA-S chart is the optimal one for the target OC value $\delta = 1.2$ in most cases. However, the CUSUM-S chart has roughly the same ARL profiles. For small shift of size $\delta = 1.1$, the best OC behavior is obtained by the EWMA-S chart and the CUSUM-S chart based on the estimators supplemented with a screening method in most cases except for diffuse mean contaminations where the CS-CUSUM-S chart performs slightly better. If larger shifts of size $\delta \geq 1.4$ occur in the process variance, the performance of the three charts are very similar.

Overall the best IC performance is obtained by the CS-CUSUM-S chart and the EWMA-S and CUSUM-S charts are more efficient in detecting small and moderate shifts in most cases. However, the three schemes exhibit similar performance with subtle differences in most cases, which indicates the influence of the types of the charts is limited.

4. Impact of different change magnitudes.

   From Figures 1-7, it is seen that when the shift is small ($\delta \leq 1.2$), the shape of the run length distribution for the case with estimated parameters is far from that with known parameters, where the difference is more pronounced in the contaminated situations. However, for larger shifts ($\delta \geq 1.4$), the performance discrepancy between the known and estimated parameters cases is diminished. It indicates that the deterioration is lessened as the magnitude of the shift gets larger.

5.3 Effect of the Phase I sample size

We have noted that in general one way to improve the performance of a Phase II control chart is to increase the amount of Phase I data. We make a few comments here in this direction. To study the effect of the Phase I sample size on the performance of the charts studied, we estimate the unknown parameters from k Phase I samples, each of size $n = 5$. The values of k considered in our study range from 50 to 3000. Note that, in all contaminated cases, we keep 5% of the data in Phase I are contaminated for each k to ensure a fair comparison. We found similar results to hold for three memory charts with estimated parameters in all contaminated cases. Therefore, to save space, only the simulation results for the EWMA-S chart with the estimator $\overline{ADMsi}$ and the different values of k are presented in Table 9. The first part of the table shows the performance of the EWMA-S chart under normality, following by the results when Phase I contains diffuse symmetric variance disturbances or localized variance disturbances for illustration. The last row in the uncontaminated normal scenario (entitled $k = \infty$) displays the performance of the EWMA-S chart when the process parameters are known, which is also regarded as a basis for comparison. We adjust the control limit parameter L for each value of k to produce an ARL_0 of 370 under normality. The values of L are also provided in Table 9. It is observed that the value of L associated with the estimated-parameter case converges to that of the known-parameter case as k increases. To obtain a more global view of the run length distribution, Figure 8 presents the IC and OC run-length CDF curves of the EWMA-S chart with both known and estimated parameters for the uncontaminated and two contaminated cases, respectively.

[Insert Table 9 about here]

[Insert Figure 8 about here]

From Table 9 and Figure 8, we can see that, when the Phase I data are uncontaminated, the performance discrepancy between the known and estimated parameters cases is diminished as the Phase I sample size k increases. In the contaminated cases, the SDRL and 90th percentile decrease, whereas the 10th and 50th percentiles increase as k increases when the process is IC or slightly OC ($\delta \leq 1.2$), except for minor sampling fluctuations. This indicates that the negative effects of the variability added by the estimation process reduce as k increases. However, as it is obvious in Figure 8, when parameters are estimated, the OC CDF curves for the early runs tend to be lower and farther from that of the known-parameter case when k becomes larger, indicating that the ability of early detection reduces as k increases in the contaminated cases. Therefore, unlike in the uncontaminated cases, larger Phase I sample sizes do not necessarily lead to a better performance under contaminated environments.

6 Conclusions and considerations

In this article, we present a comparison of three memory-type control charts with different estimators. Different parent environments are used to evaluate the performance of these charts in terms of their run length distributions. We recommend using $\overline{ADM}i$ when the dataset is likely to be contaminated by diffuse disturbances. On the other hand, we prefer $\overline{ADM}s$ when localized disturbances are likely to occur in Phase I. If it is unknown what type of contaminations are present in Phase I, the estimator $\overline{ADM}si$, which combines the advantages of the above two estimators, is considered to be the best choice for a Phase I analysis. The comparisons showed that there is no single control chart which behaves well in all environments. The best IC behavior is obtained by the CS-CUSUM-S chart. Overall the best OC performance in the contaminated cases is obtained by the EWMA-S chart based on the $\overline{ADM}si$ estimator, although the differences with the other charts are insignificant.

As noted by Hawkins and Olwell,[32] a chart tuned to be more sensitive to small shifts is affected by parameter estimation more than one tuned to large shifts and that a one-sided CUSUM chart is more severely impacted than a two-sided CUSUM chart. This is also true in general of the charts studied in this paper. The EWMA-S, CUSUM-S and CS-CUSUM-S charts considered in this paper are designed to be one-sided and sensitive to small shifts. It is observed that their performance are strongly impacted by parameter estimation when the process is IC or slightly OC, even using the adjusted control limits to achieve the specified nominal ARL_0 for each chart under uncontaminated normal environments. The estimation effect is more severe in the contaminated cases. It is worth emphasizing that there is a significant difference between the run length distribution of the memory chart with estimated parameters and the analogous case with known parameters, no matter whether the Phase I data are contaminated or not. This difference is gradually diminished when a large number of Phase I samples is used under normality, but it is not true in the contaminated cases. Therefore, these memory-type charts with estimated parameters need to be used more cautiously in practice.

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Table 1: Factors of the EWMA-S, CUSUM-S and CS-CUSUM-S charts under uncontaminated normal environment with $k = 50, n = 5$ at $ARL_0 = 370$.

		Phase I estimators										
Chart	Factor	S_p	$\overline{S}$	$\overline{IQR}$	$\overline{IQR}_{20}$	$\overline{G}$	$\overline{ADM}$	$\overline{Q_n}$	$D7$	$\overline{ADMs}$	$\overline{ADMi}$	$\overline{ADMsi}$
EWMA	L	2.289	2.268	1.924	1.786	2.262	2.250	1.975	2.230	2.256	2.187	2.185
CUSUM	h_c	1.801	1.781	1.438	1.322	1.778	1.763	1.482	1.738	1.771	1.696	1.695
CS-CUSUM ($UCL = 2.10$)	h_{cs}	1.958	1.938	1.488	1.349	1.934	1.905	1.541	1.878	1.918	1.811	1.812

Table 2: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ with ARL_0=370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	σ	369.81(361.82; 47, 259, 841)	53.84(45.40; 13, 40, 113)	20.93(14.40; 7, 17, 40)	8.79(4.57; 4, 8, 15)	4.20(1.80; 2, 4, 7)
	S_p	372.80(935.28; 19, 112, 799)	48.85(85.75; 8, 27, 104)	17.94(17.13; 5, 13, 35)	7.54(4.54; 3, 6, 13)	3.68(1.71; 2, 3, 6)
	$\overline{S}$	370.79(938.66; 18, 108, 803)	48.82(100.09; 8, 26, 101)	17.79(17.29; 5, 13, 35)	7.46(4.49; 3, 6, 13)	3.65(1.70; 2, 3, 6)
	$\overline{Q_n}$	371.40(1184.88; 11, 62, 674)	51.74(199.98; 6, 19, 96)	16.79(33.30; 4, 11, 33)	6.66(4.72; 3, 5, 12)	3.27(1.64; 2, 3, 5)
	$\overline{ADM}$	370.82(962.36; 17, 103, 789)	48.43(104.78; 8, 26, 101)	17.67(17.24; 5, 13, 35)	7.41(4.49; 3, 6, 13)	3.63(1.70; 2, 3, 6)
	$\overline{G}$	372.09(957.25; 18, 106, 792)	48.61(87.28; 8, 26, 103)	17.76(17.22; 5, 13, 35)	7.44(4.48; 3, 6, 13)	3.64(1.69; 2, 3, 6)
	$\overline{IQR}$	372.04(1222.40; 10, 56, 656)	52.45(237.34; 6, 18, 93)	16.49(34.73; 4, 10, 32)	6.49(4.68; 3, 5, 12)	3.19(1.63; 2, 3, 5)
	$\overline{IQR}_{20}$	371.54(1288.02; 8, 44, 602)	57.19(295.60; 5, 16, 90)	16.26(41.18; 3, 9, 31)	6.16(4.88; 2, 5, 11)	3.03(1.63; 1, 3, 5)
	$D7$	370.11(976.17; 17, 100, 780)	48.68(98.20; 8, 25, 102)	17.57(18.32; 5, 13, 35)	7.36(4.49; 3, 6, 13)	3.61(1.70; 2, 3, 6)
	$\overline{ADMs}$	369.03(950.73; 18, 105, 785)	48.74(97.99; 8, 26, 103)	17.76(17.52; 5, 13, 35)	7.41(4.50; 3, 6, 13)	3.63(1.69; 2, 3, 6)
	$\overline{ADMi}$	370.34(1002.48; 15, 93, 773)	48.02(113.83; 7, 24, 99)	17.40(19.15; 5, 12, 34)	7.20(4.48; 3, 6, 13)	3.55(1.67; 2, 3, 6)
	$\overline{ADMsi}$	370.20(1000.05; 15, 92, 784)	48.72(106.00; 7, 24, 102)	17.39(18.30; 5, 12, 35)	7.23(4.55; 3, 6, 13)	3.54(1.68; 2, 3, 6)
CUSUM	σ	370.40(361.44; 48, 260, 840)	52.27(43.40; 13, 39, 109)	20.70(13.72; 8, 17, 38)	8.82(4.47; 4, 8, 15)	4.25(1.79; 2, 4, 7)
	S_p	371.62(917.44; 19, 115, 805)	48.38(78.10; 8, 26, 104)	17.90(17.49; 5, 13, 35)	7.35(4.49; 3, 6, 13)	3.56(1.69; 2, 3, 6)
	$\overline{S}$	370.21(922.21; 18, 111, 799)	48.97(88.54; 8, 26, 105)	17.70(17.68; 5, 13, 35)	7.28(4.54; 3, 6, 13)	3.53(1.69; 2, 3, 6)
	$\overline{Q_n}$	369.40(1155.03; 10, 66, 695)	53.17(193.79; 6, 20, 101)	16.74(25.95; 4, 10, 33)	6.45(4.76; 2, 5, 12)	3.10(1.64; 1, 3, 5)
	$\overline{ADM}$	371.17(950.79; 17, 106, 794)	49.47(96.02; 8, 26, 104)	17.62(17.76; 5, 12, 35)	7.22(4.47; 3, 6, 13)	3.51(1.69; 2, 3, 6)
	$\overline{G}$	371.13(934.69; 18, 108, 804)	49.43(99.80; 8, 26, 104)	17.75(17.53; 5, 13, 36)	7.27(4.52; 3, 6, 13)	3.53(1.68; 2, 3, 6)
	$\overline{IQR}$	369.99(1196.09; 9, 59, 668)	55.91(247.16; 5, 19, 100)	17.00(35.28; 4, 10, 34)	6.37(5.01; 2, 5, 12)	3.04(1.63; 1, 3, 5)
	$\overline{IQR}_{20}$	371.00(1258.81; 8, 48, 636)	58.45(287.24; 5, 16, 98)	16.88(44.80; 3, 9, 32)	6.04(5.30; 2, 5, 11)	2.88(1.63; 1, 3, 5)
	$D7$	371.87(973.71; 16, 101, 786)	48.39(98.50; 7, 25, 102)	17.53(18.19; 5, 12, 35)	7.17(4.51; 3, 6, 13)	3.47(1.69; 2, 3, 6)
	$\overline{ADMs}$	371.70(920.21; 18, 109, 815)	49.17(94.24; 8, 26, 105)	17.73(18.00; 5, 13, 35)	7.28(4.52; 3, 6, 13)	3.52(1.68; 2, 3, 6)
	$\overline{ADMi}$	372.11(993.11; 15, 96, 788)	49.31(106.23; 7, 24, 104)	17.54(19.08; 5, 12, 35)	7.04(4.55; 3, 6, 13)	3.41(1.67; 2, 3, 6)
	$\overline{ADMsi}$	370.10(982.93; 15, 95, 791)	49.16(106.99; 7, 24, 105)	17.50(19.32; 5, 12, 35)	7.06(4.60; 3, 6, 13)	3.41(1.67; 2, 3, 6)
CS-CUSUM	σ	371.57(367.01; 45, 260, 847)	56.34(47.99; 12, 42, 119)	21.56(15.09; 6, 18, 41)	8.51(5.26; 2, 8, 15)	3.47(2.29; 1, 3, 7)
	S_p	371.24(847.34; 19, 122, 841)	52.64(93.90; 8, 28, 114)	18.57(18.85; 5, 13, 37)	7.22(4.89; 2, 6, 13)	3.15(1.97; 1, 3, 6)
	$\overline{S}$	371.33(867.75; 19, 119, 836)	52.88(96.31; 8, 28, 114)	18.42(18.90; 5, 13, 37)	7.21(4.91; 2, 6, 13)	3.13(1.96; 1, 3, 6)
	$\overline{Q_n}$	371.67(1129.72; 10, 68, 730)	54.29(198.26; 6, 20, 104)	17.20(29.87; 4, 10, 34)	6.38(5.00; 2, 5, 12)	2.87(1.76; 1, 3, 5)
	$\overline{ADM}$	369.87(876.25; 18, 113, 831)	52.82(102.85; 8, 27, 112)	18.18(19.23; 5, 13, 37)	7.11(4.87; 2, 6, 13)	3.12(1.93; 1, 3, 6)
	$\overline{G}$	372.04(868.95; 19, 118, 838)	52.58(97.93; 8, 27, 113)	18.50(19.00; 5, 13, 37)	7.18(4.91; 2, 6, 13)	3.14(1.96; 1, 3, 6)
	$\overline{IQR}$	370.76(1167.62; 10, 60, 703)	54.68(210.43; 5, 19, 103)	16.85(30.14; 3, 10, 34)	6.22(5.06; 2, 5, 12)	2.85(1.77; 1, 3, 5)
	$\overline{IQR}_{20}$	369.64(1249.50; 8, 49, 627)	60.21(285.04; 4, 16, 99)	17.18(46.71; 3, 9, 33)	5.98(5.31; 2, 5, 11)	2.74(1.69; 1, 2, 5)
	$D7$	369.23(899.75; 17, 110, 824)	52.31(98.65; 7, 26, 111)	18.19(19.20; 5, 13, 37)	7.09(4.92; 2, 6, 13)	3.11(1.94; 1, 3, 6)
	$\overline{ADMs}$	372.78(866.56; 18, 116, 850)	52.56(92.44; 8, 28, 112)	18.34(19.38; 5, 13, 37)	7.19(4.93; 2, 6, 13)	3.12(1.95; 1, 3, 6)
	$\overline{ADMi}$	371.99(950.91; 15, 101, 808)	51.73(107.45; 7, 25, 110)	17.85(20.19; 4, 12, 36)	6.93(4.85; 2, 6, 13)	3.07(1.90; 1, 3, 6)
	$\overline{ADMsi}$	371.02(932.75; 16, 102, 822)	51.69(105.56; 7, 25, 111)	17.90(20.46; 4, 12, 36)	6.96(4.87; 2, 6, 13)	3.06(1.91; 1, 3, 6)

Table 3: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when diffuse symmetric variance disturbances are present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	4453.34(4220.52; 115, 2489,10000)	1041.76(2299.35; 20, 161, 2863)	161.13(677.30; 9, 36, 251)	15.27(24.36; 4, 10, 29)	4.92(2.65; 2, 4, 8)
	$\overline{S}$	3146.86(3777.91; 73, 1103,10000)	480.85(1332.23; 16, 98, 976)	66.99(227.26; 8, 27, 129)	12.04(11.07; 4, 9, 22)	4.56(2.33; 2, 4, 8)
	$\overline{Q_n}$	1382.97(2704.40; 20, 208, 4991)	206.64(796.33; 8, 38, 340)	37.66(158.83; 5, 16, 69)	8.94(8.23; 3, 7, 17)	3.75(1.99; 2, 3, 6)
	$\overline{ADM}$	2482.47(3413.36; 56, 729,10000)	313.05(924.92; 14, 76, 614)	49.75(135.47; 8, 24, 99)	10.98(8.70; 4, 9, 20)	4.36(2.17; 2, 4, 7)
	$\overline{G}$	2835.07(3617.38; 66, 912,10000)	390.53(1124.14; 15, 87, 769)	56.70(167.99; 8, 25, 113)	11.45(9.43; 4, 9, 21)	4.46(2.24; 2, 4, 7)
	$\overline{IQR}$	935.48(2193.20; 14, 124, 2416)	130.79(559.60; 7, 28, 212)	27.54(87.43; 5, 13, 52)	7.96(6.75; 3, 6, 15)	3.52(1.88; 2, 3, 6)
	$\overline{IQR}_{20}$	780.97(2036.02; 11, 83, 1784)	122.83(583.21; 6, 23, 181)	27.45(148.85; 4, 11, 46)	7.38(7.17; 3, 6, 14)	3.30(1.83; 2, 3, 6)
	$\overline{D7}$	1429.77(2571.92; 33, 324, 4653)	152.02(474.19; 11, 47, 300)	31.82(56.33; 6, 18, 65)	9.33(6.67; 4, 8, 17)	4.06(2.01; 2, 4, 7)
	$\overline{ADMs}$	1914.39(3045.95; 39, 452, 8278)	237.65(781.72; 12, 58, 440)	41.16(117.40; 7, 20, 80)	10.05(7.66; 4, 8, 18)	4.21(2.11; 2, 4, 7)
	$\overline{ADMi}$	1076.74(2222.18; 23, 210, 2970)	116.10(386.77; 9, 37, 224)	27.10(54.93; 6, 16, 55)	8.67(6.21; 3, 7, 16)	3.87(1.91; 2, 3, 6)
	$\overline{ADMsi}$	1074.85(2217.19; 23, 209, 3005)	120.85(405.53; 9, 38, 233)	26.98(44.48; 6, 16, 55)	8.61(6.15; 3, 7, 16)	3.87(1.91; 2, 3, 6)
CUSUM	S_p	4394.77(4200.01; 115, 2423,10000)	1023.36(2269.10; 20, 162, 2747)	163.65(666.60; 9, 36, 259)	15.20(38.18; 4, 10, 29)	4.75(2.63; 2, 4, 8)
	$\overline{S}$	3111.61(3751.13; 74, 1089,10000)	474.35(1308.86; 16, 99, 956)	67.06(220.19; 8, 27, 130)	11.88(11.86; 4, 9, 22)	4.41(2.30; 2, 4, 7)
	$\overline{Q_n}$	1330.20(2609.72; 19, 213, 4474)	198.69(745.68; 8, 40, 339)	38.09(145.04; 5, 16, 72)	8.80(8.66; 3, 7, 17)	3.58(1.99; 2, 3, 6)
	$\overline{ADM}$	2466.25(3380.35; 57, 746,10000)	315.33(928.66; 14, 78, 624)	50.01(121.54; 7, 23, 102)	10.82(9.11; 4, 8, 20)	4.23(2.16; 2, 4, 7)
	$\overline{G}$	2809.96(3586.55; 66, 925,10000)	399.48(1141.33; 15, 90, 790)	58.41(180.66; 8, 25, 116)	11.31(9.56; 4, 9, 21)	4.33(2.24; 2, 4, 7)
	$\overline{IQR}$	911.08(2125.78; 14, 132, 2307)	135.28(557.87; 7, 29, 227)	29.11(103.99; 4, 13, 55)	7.79(7.18; 3, 6, 15)	3.36(1.87; 2, 3, 6)
	$\overline{IQR}_{20}$	779.18(2002.15; 11, 90, 1831)	129.81(595.75; 5, 24, 197)	27.95(121.97; 4, 11, 51)	7.34(7.79; 2, 5, 14)	3.14(1.84; 1, 3, 5)
	$\overline{D7}$	1386.48(2498.36; 33, 330, 4286)	152.95(478.80; 11, 48, 304)	32.04(59.10; 6, 18, 65)	9.14(6.76; 3, 7, 17)	3.91(1.97; 2, 3, 6)
	$\overline{ADMs}$	1904.60(3025.27; 40, 459, 8048)	232.41(750.68; 12, 58, 440)	41.39(120.12; 7, 20, 82)	9.96(7.92; 4, 8, 18)	4.08(2.08; 2, 4, 7)
	$\overline{ADMi}$	1059.76(2174.13; 23, 215, 2913)	122.96(417.97; 9, 38, 239)	27.51(50.76; 5, 16, 56)	8.48(6.28; 3, 7, 16)	3.71(1.89; 2, 3, 6)
	$\overline{ADMsi}$	1061.16(2184.68; 23, 216, 2909)	120.98(396.82; 9, 38, 235)	27.45(48.42; 5, 16, 56)	8.44(6.23; 3, 7, 15)	3.72(1.91; 2, 3, 6)
CS-CUSUM	S_p	3882.20(3971.32; 114, 1903,10000)	928.21(2031.38; 20, 171, 2425)	161.69(607.90; 9, 38, 278)	15.54(24.41; 4, 10, 30)	4.46(2.96; 1, 4, 8)
	$\overline{S}$	2771.65(3452.02; 79, 1033,10000)	460.94(1196.04; 16, 107, 999)	72.18(220.42; 8, 29, 143)	12.01(11.31; 3, 9, 23)	4.09(2.63; 1, 4, 7)
	$\overline{Q_n}$	1297.99(2542.91; 20, 219, 4195)	202.62(750.97; 8, 40, 356)	39.50(153.28; 5, 16, 74)	8.80(8.88; 2, 7, 17)	3.38(2.16; 1, 3, 6)
	$\overline{ADM}$	2209.45(3100.45; 60, 713, 8813)	313.87(858.87; 14, 83, 656)	53.51(129.12; 7, 25, 110)	10.91(9.46; 3, 9, 21)	3.90(2.48; 1, 4, 7)
	$\overline{G}$	2523.60(3307.16; 70, 884,10000)	380.23(1003.63; 15, 94, 825)	63.03(178.64; 7, 27, 126)	11.55(10.26; 3, 9, 22)	4.01(2.56; 1, 4, 7)
	$\overline{IQR}$	905.95(2090.19; 15, 136, 2344)	135.00(545.56; 7, 30, 232)	29.73(111.32; 4, 13, 57)	7.80(7.53; 2, 6, 15)	3.16(2.00; 1, 3, 6)
	$\overline{IQR}_{20}$	770.81(1975.19; 10, 92, 1787)	126.77(578.69; 5, 24, 197)	28.53(133.64; 4, 11, 51)	7.31(7.70; 2, 5, 14)	3.01(1.92; 1, 3, 5)
	$\overline{D7}$	1300.11(2320.53; 35, 341, 3822)	160.71(464.85; 11, 51, 332)	33.96(59.58; 6, 19, 71)	9.18(7.15; 3, 7, 17)	3.58(2.26; 1, 3, 7)
	$\overline{ADMs}$	1717.74(2752.69; 41, 467, 5913)	234.71(702.28; 12, 62, 466)	43.07(109.73; 6, 21, 86)	9.98(8.32; 3, 8, 19)	3.73(2.38; 1, 3, 7)
	$\overline{ADMi}$	987.57(2010.18; 23, 222, 2647)	122.71(362.30; 9, 40, 252)	28.60(50.16; 5, 16, 59)	8.42(6.68; 2, 7, 16)	3.40(2.15; 1, 3, 6)
	$\overline{ADMsi}$	1008.24(2041.11; 24, 225, 2714)	125.43(390.10; 9, 40, 254)	28.74(52.08; 5, 16, 59)	8.47(6.72; 3, 7, 16)	3.40(2.15; 1, 3, 6)

Table 4: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when diffuse asymmetric variance disturbances are present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	4710.89(4428.67; 82, 2635,10000)	2102.15(3615.68; 18, 178,10000)	868.87(2467.69; 9, 39, 1762)	156.85(1040.97; 4, 11, 56)	10.24(186.21; 2, 4, 10)
	$\overline{S}$	3028.05(3837.52; 54, 855,10000)	638.73(1801.92; 14, 85, 1232)	110.15(569.86; 8, 25, 148)	12.77(40.01; 4, 9, 23)	4.52(2.42; 2, 4, 7)
	$\overline{Q_n}$	954.21(2198.63; 16, 138, 2493)	132.95(563.07; 7, 30, 214)	27.11(69.41; 5, 14, 52)	8.09(6.69; 3, 6, 15)	3.58(1.87; 2, 3, 6)
	$\overline{ADM}$	2242.50(3335.65; 44, 542,10000)	331.05(1094.82; 13, 64, 575)	53.32(220.36; 7, 22, 95)	10.73(10.01; 4, 8, 20)	4.29(2.18; 2, 4, 7)
	$\overline{G}$	2583.54(3568.09; 49, 664,10000)	443.14(1383.57; 13, 73, 778)	68.65(323.21; 7, 23, 111)	11.36(12.34; 4, 9, 21)	4.39(2.26; 2, 4, 7)
	$\overline{IQR}$	733.27(1913.72; 12, 95, 1618)	96.73(425.47; 6, 24, 159)	23.94(107.65; 4, 12, 44)	7.44(6.14; 3, 6, 14)	3.42(1.80; 2, 3, 6)
	$\overline{IQR}_{20}$	614.04(1771.89; 10, 65, 1215)	89.87(449.69; 5, 20, 136)	22.51(93.58; 4, 10, 41)	6.89(6.28; 2, 5, 13)	3.20(1.74; 2, 3, 5)
	$D7$	845.23(1844.64; 24, 191, 2074)	90.18(255.74; 9, 35, 182)	24.06(32.22; 6, 15, 48)	8.39(5.62; 3, 7, 15)	3.85(1.85; 2, 3, 6)
	$\overline{ADMs}$	1034.38(2132.89; 26, 222, 2758)	109.68(364.88; 10, 38, 210)	26.62(49.27; 6, 16, 53)	8.69(5.97; 3, 7, 16)	3.92(1.90; 2, 4, 6)
	$\overline{ADMi}$	688.50(1649.09; 19, 141, 1578)	75.63(230.94; 8, 30, 152)	21.63(30.00; 5, 14, 44)	7.91(5.22; 3, 7, 14)	3.71(1.79; 2, 3, 6)
	$\overline{ADMsi}$	672.95(1619.36; 19, 141, 1530)	76.52(233.24; 8, 30, 154)	21.80(29.24; 5, 14, 44)	7.94(5.29; 3, 7, 14)	3.71(1.79; 2, 3, 6)
CUSUM	S_p	4694.78(4417.21; 82, 2638,10000)	2067.08(3583.34; 17, 180,10000)	865.61(2460.42; 9, 39, 1749)	158.33(1038.55; 4, 11, 57)	11.48(208.93; 2, 4, 10)
	$\overline{S}$	2992.36(3810.56; 55, 849,10000)	635.98(1779.15; 14, 86, 1240)	106.76(540.89; 7, 25, 146)	12.68(29.17; 4, 9, 23)	4.40(2.41; 2, 4, 7)
	$\overline{Q_n}$	930.20(2133.10; 15, 141, 2413)	128.14(511.20; 7, 31, 221)	28.51(90.61; 4, 14, 55)	7.91(7.15; 3, 6, 15)	3.41(1.86; 2, 3, 6)
	$\overline{ADM}$	2215.95(3296.08; 44, 547,10000)	326.55(1056.79; 12, 65, 582)	53.66(218.28; 7, 21, 96)	10.52(10.56; 4, 8, 20)	4.16(2.16; 2, 4, 7)
	$\overline{G}$	2580.91(3552.82; 50, 686,10000)	436.41(1342.35; 13, 74, 783)	72.00(347.78; 7, 23, 114)	11.37(33.88; 4, 8, 21)	4.26(2.24; 2, 4, 7)
	$\overline{IQR}$	703.05(1820.57; 12, 100, 1574)	101.27(438.80; 6, 25, 169)	24.48(83.36; 4, 12, 47)	7.27(7.72; 2, 6, 14)	3.26(1.80; 2, 3, 6)
	$\overline{IQR}_{20}$	600.40(1710.09; 9, 71, 1212)	98.25(473.09; 5, 21, 153)	23.26(93.63; 4, 10, 43)	6.83(6.97; 2, 5, 13)	3.04(1.76; 1, 3, 5)
	$D7$	837.61(1819.09; 24, 192, 2058)	90.48(243.61; 9, 36, 188)	24.12(32.44; 6, 15, 49)	8.19(5.64; 3, 7, 15)	3.72(1.84; 2, 3, 6)
	$\overline{ADMs}$	1037.55(2120.11; 26, 227, 2769)	111.21(364.87; 9, 39, 217)	26.69(53.81; 6, 16, 54)	8.55(6.01; 3, 7, 15)	3.81(1.88; 2, 3, 6)
	$\overline{ADMi}$	690.71(1630.01; 19, 147, 1595)	76.59(213.32; 8, 30, 157)	21.83(29.90; 5, 14, 44)	7.72(5.30; 3, 6, 14)	3.57(1.78; 2, 3, 6)
	$\overline{ADMsi}$	668.66(1585.68; 19, 147, 1535)	76.87(210.24; 8, 30, 158)	22.01(31.50; 5, 14, 44)	7.74(5.35; 3, 6, 14)	3.57(1.79; 2, 3, 6)
CS-CUSUM	S_p	4315.46(4280.43; 83, 2097,10000)	1894.02(3379.96; 18, 184,10000)	798.12(2303.40; 9, 42, 1578)	147.76(971.53; 4, 11, 61)	10.51(185.00; 1, 4, 10)
	$\overline{S}$	2692.11(3545.08; 57, 816,10000)	571.23(1577.08; 14, 92, 1170)	110.62(524.50; 7, 26, 160)	13.11(35.90; 3, 9, 24)	4.07(2.74; 1, 4, 7)
	$\overline{Q_n}$	907.77(2068.55; 15, 144, 2373)	138.43(556.83; 7, 31, 235)	29.26(82.78; 4, 14, 57)	7.81(7.03; 2, 6, 15)	3.21(2.03; 1, 3, 6)
	$\overline{ADM}$	1996.49(3038.88; 45, 536, 8139)	319.16(984.15; 13, 70, 615)	56.05(207.41; 7, 22, 102)	10.58(10.32; 3, 8, 20)	3.80(2.46; 1, 3, 7)
	$\overline{G}$	2319.92(3277.89; 52, 666,10000)	421.03(1237.34; 14, 81, 821)	73.45(317.44; 7, 25, 125)	11.44(13.26; 3, 9, 22)	3.93(2.57; 1, 4, 7)
	$\overline{IQR}$	703.87(1806.08; 12, 103, 1585)	102.60(431.42; 6, 25, 177)	24.88(79.22; 4, 12, 48)	7.21(6.56; 2, 6, 14)	3.06(1.92; 1, 3, 5)
	$\overline{IQR}_{20}$	610.74(1725.04; 9, 72, 1241)	96.56(447.38; 5, 21, 155)	23.44(85.93; 3, 10, 44)	6.76(6.66; 2, 5, 13)	2.91(1.84; 1, 3, 5)
	$D7$	804.54(1684.40; 25, 206, 2000)	97.90(264.66; 9, 38, 200)	25.57(39.65; 5, 16, 52)	8.21(6.13; 2, 7, 15)	3.38(2.12; 1, 3, 6)
	$\overline{ADMs}$	950.81(1913.24; 27, 235, 2486)	116.99(338.77; 10, 42, 240)	28.11(48.36; 6, 17, 57)	8.54(6.52; 3, 7, 16)	3.44(2.17; 1, 3, 6)
	$\overline{ADMi}$	649.23(1489.57; 19, 153, 1548)	79.84(212.21; 8, 32, 167)	22.59(34.62; 5, 14, 47)	7.64(5.66; 2, 6, 14)	3.23(2.02; 1, 3, 6)
	$\overline{ADMsi}$	647.90(1486.27; 19, 156, 1538)	81.44(224.18; 8, 32, 170)	22.54(31.13; 5, 14, 46)	7.68(5.68; 2, 6, 14)	3.24(2.02; 1, 3, 6)

Table 5: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when diffuse mean disturbances are present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	5279.15(4183.69; 224, 4558,10000)	1055.45(2133.36; 28, 236, 2817)	130.45(387.46; 11, 44, 263)	15.47(15.30; 5, 11, 30)	5.08(2.65; 2, 4, 8)
	$\overline{S}$	4417.46(4103.19; 150, 2620,10000)	746.79(1699.52; 22, 167, 1775)	94.09(271.18; 10, 36, 190)	13.84(12.31; 5, 10, 26)	4.83(2.48; 2, 4, 8)
	$\overline{Q_n}$	2337.31(3475.37; 32, 480,10000)	420.73(1334.08; 10, 62, 753)	66.36(282.64; 6, 21, 116)	10.83(13.40; 3, 8, 21)	4.08(2.25; 2, 4, 7)
	$\overline{ADM}$	3810.47(3967.98; 114, 1791,10000)	566.32(1398.62; 19, 132, 1252)	77.00(220.99; 9, 32, 155)	12.84(10.89; 4, 10, 24)	4.69(2.40; 2, 4, 8)
	$\overline{G}$	4152.02(4050.83; 136, 2214,10000)	659.07(1554.57; 21, 151, 1502)	84.23(235.51; 9, 34, 170)	13.38(11.42; 4, 10, 26)	4.77(2.44; 2, 4, 8)
	$\overline{IQR}$	1707.03(3028.93; 21, 263, 7934)	290.83(1042.83; 9, 44, 482)	49.34(223.29; 5, 17, 85)	9.58(10.65; 3, 7, 18)	3.82(2.10; 2, 3, 6)
	$\overline{IQR}_{20}$	1283.18(2680.61; 14, 146, 4561)	233.70(957.05; 7, 31, 342)	44.05(230.89; 4, 14, 70)	8.67(10.03; 3, 6, 16)	3.54(2.00; 2, 3, 6)
	$D7$	3065.14(3721.26; 71, 1074,10000)	440.24(1214.35; 16, 96, 889)	61.91(176.60; 8, 27, 125)	11.76(9.79; 4, 9, 22)	4.49(2.29; 2, 4, 7)
	$\overline{ADMs}$	4037.72(4062.90; 112, 2007,10000)	684.79(1640.09; 20, 145, 1571)	87.49(271.83; 9, 33, 173)	13.31(12.00; 4, 10, 25)	4.73(2.43; 2, 4, 8)
	$\overline{ADMi}$	2533.65(3522.45; 43, 652,10000)	364.63(1109.82; 13, 71, 709)	55.47(187.64; 7, 23, 107)	10.92(9.39; 4, 8, 20)	4.31(2.23; 2, 4, 7)
	$\overline{ADMsi}$	2521.11(3513.47; 43, 649,10000)	365.56(1120.71; 13, 71, 696)	55.09(177.76; 7, 23, 106)	10.91(9.39; 4, 8, 20)	4.29(2.21; 2, 4, 7)
CUSUM	S_p	5185.11(4175.14; 222, 4293,10000)	1047.84(2098.94; 28, 241, 2829)	132.82(405.76; 10, 45, 268)	15.30(15.29; 5, 11, 30)	4.92(2.62; 2, 4, 8)
	$\overline{S}$	4348.88(4078.57; 151, 2524,10000)	739.04(1678.46; 22, 168, 1751)	94.91(274.32; 9, 37, 192)	13.62(12.52; 4, 10, 26)	4.69(2.48; 2, 4, 8)
	$\overline{Q_n}$	2244.93(3385.39; 31, 477,10000)	394.78(1226.95; 10, 65, 746)	66.16(277.55; 6, 21, 119)	10.81(12.56; 3, 8, 21)	3.90(2.25; 2, 3, 7)
	$\overline{ADM}$	3761.09(3939.14; 113, 1757,10000)	567.52(1387.60; 19, 134, 1271)	78.80(221.47; 9, 32, 162)	12.70(11.38; 4, 10, 24)	4.54(2.37; 2, 4, 8)
	$\overline{G}$	4111.56(4021.36; 137, 2218,10000)	664.70(1560.58; 21, 156, 1500)	86.30(237.05; 9, 35, 176)	13.27(11.61; 4, 10, 26)	4.63(2.43; 2, 4, 8)
	$\overline{IQR}$	1645.16(2945.11; 21, 269, 6988)	286.57(1023.63; 8, 46, 486)	50.47(206.83; 5, 17, 91)	9.55(13.49; 3, 7, 18)	3.64(2.10; 2, 3, 6)
	$\overline{IQR}_{20}$	1267.29(2635.34; 14, 157, 4362)	233.09(921.18; 6, 34, 363)	44.95(216.67; 4, 14, 76)	8.69(12.53; 2, 6, 17)	3.37(2.04; 1, 3, 6)
	$D7$	3008.18(3679.96; 73, 1052,10000)	427.45(1160.33; 15, 98, 892)	63.06(169.14; 8, 27, 128)	11.60(9.99; 4, 9, 22)	4.34(2.28; 2, 4, 7)
	$\overline{ADMs}$	4011.26(4040.80; 114, 1999,10000)	676.97(1609.43; 19, 146, 1569)	89.55(265.31; 9, 34, 180)	13.25(12.53; 4, 10, 25)	4.61(2.43; 2, 4, 8)
	$\overline{ADMi}$	2453.67(3456.61; 43, 636,10000)	364.58(1095.17; 12, 73, 712)	56.13(178.00; 7, 23, 110)	10.78(9.56; 4, 8, 21)	4.14(2.22; 2, 4, 7)
	$\overline{ADMsi}$	2476.13(3465.32; 43, 649,10000)	366.33(1100.83; 12, 74, 711)	56.64(190.59; 7, 23, 109)	10.81(9.73; 4, 8, 21)	4.14(2.19; 2, 4, 7)
CS-CUSUM	S_p	4556.37(3994.64; 204, 3077,10000)	959.41(1882.25; 28, 251, 2508)	135.65(368.25; 11, 48, 282)	15.68(15.57; 4, 11, 31)	4.61(2.97; 1, 4, 8)
	$\overline{S}$	3856.34(3842.91; 151, 2086,10000)	697.92(1514.86; 23, 180, 1687)	101.82(283.36; 10, 39, 209)	14.02(13.14; 4, 11, 28)	4.38(2.81; 1, 4, 8)
	$\overline{Q_n}$	2199.92(3317.65; 33, 493,10000)	402.92(1230.67; 10, 67, 781)	70.31(296.29; 6, 22, 126)	10.86(14.37; 3, 8, 21)	3.71(2.41; 1, 3, 7)
	$\overline{ADM}$	3327.69(3678.60; 113, 1510,10000)	550.43(1284.25; 20, 145, 1260)	81.45(202.85; 9, 34, 172)	13.01(11.95; 4, 10, 25)	4.23(2.70; 1, 4, 8)
	$\overline{G}$	3653.70(3775.40; 138, 1858,10000)	636.53(1421.75; 22, 166, 1494)	93.41(243.91; 9, 37, 196)	13.62(12.71; 4, 10, 27)	4.32(2.76; 1, 4, 8)
	$\overline{IQR}$	1640.89(2922.82; 21, 278, 6826)	287.57(994.11; 8, 47, 513)	53.04(219.90; 5, 17, 94)	9.53(10.56; 3, 7, 19)	3.48(2.25; 1, 3, 6)
	$\overline{IQR}_{20}$	1271.68(2626.14; 14, 160, 4378)	233.73(927.44; 6, 34, 363)	45.55(208.99; 4, 14, 79)	8.66(13.52; 2, 6, 17)	3.25(2.13; 1, 3, 6)
	$D7$	2723.41(3425.78; 73, 1004,10000)	420.35(1083.40; 16, 105, 915)	65.81(174.42; 8, 28, 135)	11.81(10.64; 3, 9, 23)	4.02(2.56; 1, 4, 7)
	$\overline{ADMs}$	3572.81(3791.86; 113, 1721,10000)	649.14(1483.11; 20, 154, 1543)	93.66(253.75; 9, 36, 195)	13.45(12.99; 4, 10, 26)	4.28(2.75; 1, 4, 8)
	$\overline{ADMi}$	2268.76(3249.28; 43, 633,10000)	351.45(996.68; 12, 78, 737)	58.20(174.73; 7, 24, 118)	10.93(10.36; 3, 8, 21)	3.83(2.46; 1, 3, 7)
	$\overline{ADMsi}$	2273.56(3253.88; 44, 637,10000)	351.87(1010.80; 12, 76, 725)	59.09(171.41; 7, 24, 118)	10.98(11.35; 3, 8, 21)	3.83(2.46; 1, 3, 7)

Table 6: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when localized variance disturbances are present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	4322.48(4353.15; 67, 1936,10000)	1450.84(2946.89; 16, 139, 6762)	352.14(1361.16; 8, 34, 442)	21.63(126.12; 4, 10, 34)	5.02(3.14; 2, 4, 8)
	$\overline{S}$	2455.57(3507.37; 44, 582,10000)	393.25(1259.50; 13, 67, 689)	61.69(279.56; 7, 22, 104)	11.03(10.80; 4, 8, 20)	4.35(2.25; 2, 4, 7)
	$\overline{Q_n}$	1849.40(3223.00; 20, 254,10000)	371.35(1323.36; 9, 43, 569)	71.53(421.03; 5, 17, 94)	10.11(13.48; 3, 7, 19)	3.89(2.18; 2, 3, 7)
	$\overline{ADM}$	2407.13(3486.59; 42, 557,10000)	401.78(1292.91; 12, 65, 687)	62.68(299.10; 7, 22, 103)	10.95(13.45; 4, 8, 20)	4.31(2.21; 2, 4, 7)
	$\overline{G}$	2436.18(3496.58; 43, 579,10000)	391.38(1255.16; 13, 67, 683)	61.38(270.37; 7, 22, 104)	11.06(11.30; 4, 8, 20)	4.33(2.23; 2, 4, 7)
	$\overline{IQR}$	1766.50(3177.67; 17, 213,10000)	375.25(1363.84; 8, 39, 547)	72.36(438.19; 5, 16, 93)	9.99(18.01; 3, 7, 18)	3.80(2.21; 2, 3, 6)
	$\overline{IQR}_{20}$	756.43(2015.93; 10, 77, 1681)	119.80(580.75; 6, 22, 170)	26.81(143.27; 4, 11, 46)	7.23(7.96; 2, 5, 13)	3.27(1.81; 2, 3, 5)
	$D7$	1009.20(2134.80; 24, 202, 2648)	115.45(418.16; 10, 37, 209)	26.63(52.09; 6, 16, 53)	8.63(6.04; 3, 7, 15)	3.90(1.91; 2, 4, 6)
	$\overline{ADMs}$	708.30(1684.80; 21, 152, 1604)	80.77(270.59; 9, 31, 155)	22.23(37.24; 6, 14, 44)	8.08(5.31; 3, 7, 14)	3.79(1.81; 2, 3, 6)
	$\overline{ADMi}$	864.08(1956.59; 20, 164, 2126)	98.00(351.50; 9, 33, 182)	24.37(44.65; 6, 15, 48)	8.24(5.83; 3, 7, 15)	3.78(1.85; 2, 3, 6)
	$\overline{ADMsi}$	753.62(1787.32; 19, 145, 1748)	86.40(302.70; 8, 31, 164)	23.06(49.18; 5, 14, 45)	8.04(5.46; 3, 7, 14)	3.73(1.83; 2, 3, 6)
CUSUM	S_p	4268.23(4330.19; 67, 1881,10000)	1447.69(2931.36; 16, 142, 6638)	349.16(1340.36; 8, 34, 455)	22.28(139.18; 4, 10, 34)	4.84(3.01; 2, 4, 8)
	$\overline{S}$	2438.56(3486.19; 44, 595,10000)	392.24(1240.32; 12, 68, 696)	61.23(259.19; 7, 22, 106)	10.90(13.09; 4, 8, 20)	4.22(2.23; 2, 4, 7)
	$\overline{Q_n}$	1786.05(3140.90; 20, 255, 9518)	364.80(1283.94; 8, 45, 579)	70.00(394.13; 5, 17, 99)	10.07(15.31; 3, 7, 19)	3.71(2.21; 2, 3, 6)
	$\overline{ADM}$	2381.94(3450.38; 42, 564,10000)	391.18(1244.35; 12, 67, 706)	63.54(284.20; 7, 22, 108)	10.82(11.30; 4, 8, 20)	4.16(2.20; 2, 4, 7)
	$\overline{G}$	2412.88(3464.94; 44, 583,10000)	397.79(1264.48; 12, 68, 705)	62.93(280.25; 7, 22, 105)	10.91(15.13; 4, 8, 20)	4.22(2.21; 2, 4, 7)
	$\overline{IQR}$	1691.59(3078.79; 17, 219, 8568)	375.47(1347.84; 7, 41, 568)	72.00(415.56; 5, 16, 96)	10.21(33.09; 3, 7, 19)	3.63(2.19; 2, 3, 6)
	$\overline{IQR}_{20}$	738.22(1946.37; 10, 82, 1689)	122.29(570.44; 5, 23, 186)	27.63(127.42; 4, 11, 49)	7.27(10.29; 2, 5, 14)	3.11(1.83; 1, 3, 5)
	$D7$	996.80(2101.10; 24, 205, 2614)	115.04(409.22; 9, 37, 213)	26.72(60.33; 6, 15, 54)	8.41(6.22; 3, 7, 15)	3.76(1.88; 2, 3, 6)
	$\overline{ADMs}$	704.82(1667.95; 21, 154, 1585)	81.68(268.60; 9, 31, 159)	22.35(36.09; 5, 14, 45)	7.93(5.36; 3, 7, 14)	3.67(1.81; 2, 3, 6)
	$\overline{ADMi}$	866.27(1953.81; 20, 167, 2128)	100.56(350.18; 8, 33, 190)	24.60(54.12; 5, 14, 49)	8.05(5.86; 3, 6, 15)	3.63(1.83; 2, 3, 6)
	$\overline{ADMsi}$	758.46(1774.69; 19, 151, 1787)	88.33(306.33; 8, 31, 170)	23.08(39.37; 5, 14, 46)	7.85(5.64; 3, 6, 14)	3.59(1.81; 2, 3, 6)
CS-CUSUM	S_p	3867.72(4136.55; 71, 1592,10000)	1289.87(2675.58; 16, 149, 4613)	314.27(1202.88; 8, 35, 446)	22.99(136.62; 3, 10, 35)	4.56(3.52; 1, 4, 8)
	$\overline{S}$	2182.22(3201.37; 46, 586, 9918)	376.34(1128.31; 13, 73, 733)	66.05(273.20; 7, 23, 116)	11.02(11.28; 3, 8, 21)	3.88(2.53; 1, 3, 7)
	$\overline{Q_n}$	1751.84(3077.95; 20, 268, 8533)	369.55(1281.31; 8, 46, 602)	71.05(403.15; 5, 17, 101)	10.12(18.21; 3, 7, 19)	3.52(2.36; 1, 3, 6)
	$\overline{ADM}$	2141.76(3187.52; 43, 549, 9692)	378.57(1162.13; 12, 70, 719)	65.90(273.76; 7, 23, 114)	11.02(12.36; 3, 8, 21)	3.84(2.50; 1, 3, 7)
	$\overline{G}$	2180.62(3197.10; 46, 585, 9860)	382.17(1155.28; 13, 73, 735)	65.73(253.85; 7, 23, 117)	11.00(11.99; 3, 8, 21)	3.87(2.52; 1, 3, 7)
	$\overline{IQR}$	1693.49(3064.12; 17, 229, 8353)	364.99(1300.01; 7, 42, 573)	74.42(418.21; 5, 16, 102)	9.97(18.45; 2, 7, 19)	3.44(2.31; 1, 3, 6)
	$\overline{IQR}_{20}$	739.58(1939.46; 10, 84, 1677)	126.46(596.46; 5, 23, 189)	26.90(105.70; 3, 11, 49)	7.15(7.95; 2, 5, 14)	2.97(1.89; 1, 3, 5)
	$D7$	945.62(1951.87; 25, 216, 2458)	118.81(379.05; 9, 39, 232)	27.84(54.05; 5, 16, 57)	8.43(6.56; 2, 7, 16)	3.40(2.15; 1, 3, 6)
	$\overline{ADMs}$	668.95(1519.03; 21, 164, 1574)	85.80(269.49; 9, 33, 169)	23.50(39.29; 5, 15, 47)	7.85(5.80; 2, 7, 15)	3.28(2.07; 1, 3, 6)
	$\overline{ADMi}$	806.58(1779.54; 21, 175, 2006)	104.15(348.78; 8, 34, 202)	25.69(49.68; 5, 15, 52)	7.98(6.18; 2, 7, 15)	3.30(2.09; 1, 3, 6)
	$\overline{ADMsi}$	718.48(1649.56; 19, 158, 1708)	91.12(294.74; 8, 32, 179)	24.07(51.60; 5, 14, 49)	7.83(6.02; 2, 6, 15)	3.25(2.04; 1, 3, 6)

Table 7: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when a single step shift in the variance is present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	5473.53(4266.84; 197, 5316,10000)	1515.86(2791.31; 27, 270, 5703)	245.29(899.51; 11, 48, 396)	17.96(31.88; 5, 12, 34)	5.20(2.85; 2, 5, 9)
	$\overline{S}$	2789.35(3514.70; 77, 978,10000)	313.50(842.33; 16, 88, 653)	50.16(105.04; 8, 26, 104)	11.32(8.61; 4, 9, 21)	4.45(2.20; 2, 4, 7)
	$\overline{Q_n}$	2054.86(3288.54; 27, 380,10000)	349.92(1173.65; 10, 54, 604)	56.39(238.59; 6, 19, 99)	10.33(11.06; 3, 8, 19)	3.99(2.19; 2, 4, 7)
	$\overline{ADM}$	2712.96(3494.93; 71, 906,10000)	327.58(911.21; 15, 87, 672)	50.80(118.73; 8, 25, 105)	11.26(8.77; 4, 9, 21)	4.43(2.20; 2, 4, 7)
	$\overline{G}$	2770.58(3503.76; 74, 958,10000)	323.18(877.57; 16, 88, 676)	50.49(111.49; 8, 26, 105)	11.26(8.55; 4, 9, 21)	4.44(2.21; 2, 4, 7)
	$\overline{IQR}$	1949.62(3244.42; 23, 314,10000)	358.40(1235.08; 9, 49, 596)	59.19(282.69; 5, 18, 97)	10.11(11.79; 3, 7, 19)	3.92(2.19; 2, 3, 7)
	$\overline{IQR_{20}}$	801.00(2062.67; 11, 87, 1878)	123.37(585.92; 6, 23, 180)	27.12(127.40; 4, 11, 48)	7.42(7.01; 3, 6, 14)	3.30(1.81; 2, 3, 6)
	$\overline{D7}$	1065.15(2134.98; 27, 239, 2898)	110.16(324.33; 10, 40, 223)	26.78(40.57; 6, 17, 55)	8.79(6.05; 3, 7, 16)	3.93(1.91; 2, 4, 6)
	$\overline{ADMs}$	729.66(1674.06; 22, 163, 1724)	81.45(246.45; 9, 33, 163)	22.55(29.22; 6, 15, 45)	8.17(5.33; 3, 7, 14)	3.81(1.81; 2, 3, 6)
	$\overline{ADMi}$	905.60(1967.87; 21, 185, 2315)	97.85(294.23; 9, 35, 195)	24.76(38.03; 6, 15, 51)	8.36(5.75; 3, 7, 15)	3.82(1.88; 2, 3, 6)
	$\overline{ADMsi}$	783.30(1790.22; 20, 158, 1896)	86.05(264.90; 9, 32, 172)	23.01(31.28; 5, 14, 47)	8.14(5.54; 3, 7, 15)	3.75(1.83; 2, 3, 6)
CUSUM	S_p	5403.56(4256.25; 201, 4982,10000)	1457.85(2720.29; 27, 265, 5187)	240.23(885.16; 11, 48, 392)	17.79(36.48; 5, 11, 34)	5.05(2.84; 2, 4, 9)
	$\overline{S}$	2718.68(3459.40; 77, 958,10000)	324.38(876.02; 16, 92, 678)	50.86(99.58; 8, 26, 108)	11.10(8.59; 4, 9, 21)	4.32(2.19; 2, 4, 7)
	$\overline{Q_n}$	1973.36(3200.78; 27, 376,10000)	343.36(1128.58; 9, 56, 613)	59.75(250.96; 5, 19, 106)	10.23(12.12; 3, 7, 20)	3.80(2.18; 2, 3, 7)
	$\overline{ADM}$	2672.70(3452.19; 72, 901,10000)	322.85(877.90; 15, 89, 668)	51.62(130.07; 8, 25, 107)	11.07(8.86; 4, 9, 21)	4.30(2.21; 2, 4, 7)
	$\overline{G}$	2714.10(3466.76; 75, 946,10000)	319.53(855.13; 16, 90, 677)	51.40(110.79; 8, 26, 108)	11.13(8.72; 4, 9, 21)	4.32(2.19; 2, 4, 7)
	$\overline{IQR}$	1866.21(3143.06; 23, 322, 9409)	346.30(1164.25; 9, 51, 610)	59.95(257.82; 5, 18, 105)	10.09(12.69; 3, 7, 19)	3.73(2.17; 2, 3, 6)
	$\overline{IQR_{20}}$	789.64(2011.03; 11, 93, 1867)	125.92(566.20; 6, 24, 197)	28.79(140.83; 4, 11, 51)	7.37(7.61; 2, 5, 14)	3.16(1.85; 1, 3, 5)
	$\overline{D7}$	1049.29(2097.72; 27, 243, 2805)	111.86(330.98; 10, 41, 229)	27.09(42.02; 6, 16, 56)	8.58(6.00; 3, 7, 16)	3.79(1.89; 2, 3, 6)
	$\overline{ADMs}$	735.23(1679.82; 22, 167, 1724)	81.10(222.81; 9, 33, 166)	22.50(28.79; 5, 15, 46)	8.03(5.36; 3, 7, 14)	3.69(1.81; 2, 3, 6)
	$\overline{ADMi}$	902.28(1944.21; 22, 192, 2298)	99.64(304.66; 9, 35, 203)	25.27(41.07; 5, 15, 52)	8.18(5.86; 3, 7, 15)	3.67(1.85; 2, 3, 6)
	$\overline{ADMsi}$	779.18(1763.63; 20, 166, 1882)	86.97(258.00; 8, 33, 177)	23.24(33.44; 5, 14, 48)	7.97(5.63; 3, 6, 14)	3.62(1.82; 2, 3, 6)
CS-CUSUM	S_p	4787.38(4115.88; 193, 3392,10000)	1321.80(2469.71; 28, 277, 4127)	235.52(802.92; 11, 51, 425)	18.52(36.03; 4, 12, 36)	4.75(3.17; 1, 4, 9)
	$\overline{S}$	2416.57(3158.92; 79, 911, 9380)	325.36(810.83; 17, 100, 712)	54.29(110.96; 8, 27, 114)	11.33(9.28; 3, 9, 22)	3.98(2.53; 1, 4, 7)
	$\overline{Q_n}$	1958.33(3148.92; 29, 394, 9328)	343.89(1108.70; 10, 58, 634)	60.39(243.88; 5, 20, 109)	10.26(11.84; 3, 7, 20)	3.62(2.34; 1, 3, 7)
	$\overline{ADM}$	2381.45(3160.16; 73, 864, 9389)	324.81(829.06; 16, 95, 713)	54.10(109.93; 8, 27, 115)	11.22(9.39; 3, 9, 22)	3.96(2.50; 1, 4, 7)
	$\overline{G}$	2415.18(3162.82; 77, 907, 9445)	328.67(810.37; 16, 99, 727)	53.80(106.73; 8, 27, 115)	11.31(9.32; 3, 9, 22)	3.99(2.51; 1, 4, 7)
	$\overline{IQR}$	1868.27(3119.81; 24, 334, 9021)	349.08(1158.43; 9, 53, 628)	61.16(264.14; 5, 19, 107)	10.08(12.87; 3, 7, 20)	3.53(2.31; 1, 3, 6)
	$\overline{IQR_{20}}$	796.19(2013.10; 11, 95, 1896)	132.70(608.03; 5, 25, 204)	28.78(115.43; 4, 12, 53)	7.33(8.06; 2, 5, 14)	3.02(1.93; 1, 3, 5)
	$\overline{D7}$	984.27(1922.59; 28, 255, 2589)	116.59(308.76; 10, 43, 246)	28.46(42.59; 6, 17, 59)	8.59(6.49; 3, 7, 16)	3.46(2.16; 1, 3, 6)
	$\overline{ADMs}$	701.54(1536.99; 22, 178, 1704)	85.81(223.05; 9, 35, 178)	23.70(35.71; 5, 15, 48)	7.94(5.80; 2, 7, 15)	3.32(2.09; 1, 3, 6)
	$\overline{ADMi}$	854.23(1807.34; 22, 199, 2157)	102.45(282.37; 9, 37, 213)	25.90(38.93; 5, 15, 54)	8.11(6.26; 2, 7, 15)	3.34(2.09; 1, 3, 6)
	$\overline{ADMsi}$	754.37(1659.65; 20, 174, 1863)	91.48(255.53; 8, 34, 190)	24.15(34.71; 5, 15, 50)	7.92(5.96; 2, 7, 15)	3.29(2.06; 1, 3, 6)

Table 8: Performance comparisons between the CUSUM-S, EWMA-S and CS-CUSUM-S charts for $k = 50, n = 5$ when multiple step shifts in the variance are present in Phase I. The control limits used are those producing an ARL_0 of 370 under uncontaminated normal environment.

		ARL(SDRL; 10^{th}, 50^{th}, 90^{th} percentiles) of the unconditional run length distribution				
chart	estimator	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
EWMA	S_p	4167.20(4509.65; 38, 1155,10000)	1981.50(3568.49; 13, 106,10000)	763.21(2277.45; 7, 29, 1385)	72.84(576.68; 4, 10, 50)	5.35(5.98; 2, 4, 9)
	$\overline{S}$	2787.90(3882.46; 32, 494,10000)	777.71(2161.29; 11, 63, 1588)	171.61(898.24; 7, 22, 169)	14.62(79.29; 4, 8, 23)	4.47(2.59; 2, 4, 7)
	$\overline{Q_n}$	2157.66(3576.74; 17, 231,10000)	632.98(1979.35; 8, 40, 1027)	161.48(898.03; 5, 17, 135)	15.30(144.01; 3, 7, 21)	4.00(2.58; 2, 3, 7)
	$\overline{ADM}$	2752.65(3868.81; 31, 472,10000)	763.39(2146.68; 11, 60, 1522)	172.35(901.88; 7, 21, 166)	15.10(100.32; 4, 8, 23)	4.42(2.60; 2, 4, 7)
	$\overline{G}$	2770.89(3871.57; 32, 489,10000)	771.44(2147.80; 11, 62, 1583)	174.59(910.25; 7, 21, 169)	14.82(93.17; 4, 8, 23)	4.44(2.81; 2, 4, 7)
	$\overline{IQR}$	2016.48(3479.10; 15, 195,10000)	616.97(1959.67; 7, 37, 977)	158.85(898.15; 5, 16, 129)	14.89(135.39; 3, 7, 21)	3.92(2.59; 2, 3, 7)
	$\overline{IQR}_{20}$	874.71(2239.47; 10, 78, 2110)	162.64(805.15; 6, 22, 195)	34.92(245.88; 4, 11, 49)	7.61(16.42; 2, 5, 14)	3.30(1.87; 2, 3, 6)
	$\overline{D7}$	1263.15(2569.78; 23, 203, 4029)	196.41(849.20; 9, 37, 270)	39.55(248.53; 6, 16, 59)	9.07(12.95; 3, 7, 16)	3.94(1.98; 2, 4, 6)
	$\overline{ADMs}$	825.79(1955.47; 21, 152, 1904)	112.53(536.46; 9, 32, 169)	28.12(168.75; 6, 14, 46)	8.34(9.20; 3, 7, 15)	3.83(1.86; 2, 3, 6)
	$\overline{ADMi}$	1027.21(2280.50; 19, 163, 2768)	149.21(670.35; 9, 33, 217)	31.11(161.42; 5, 15, 52)	8.52(8.67; 3, 7, 15)	3.82(1.92; 2, 3, 6)
	$\overline{ADMsi}$	899.26(2091.13; 19, 149, 2180)	124.73(572.45; 8, 31, 188)	27.48(113.00; 5, 14, 49)	8.28(7.44; 3, 7, 15)	3.76(1.87; 2, 3, 6)
CUSUM	S_p	4151.34(4496.66; 38, 1171,10000)	1958.27(3537.22; 12, 107,10000)	745.72(2239.61; 7, 30, 1353)	72.71(580.31; 4, 9, 49)	5.23(5.85; 2, 4, 9)
	$\overline{S}$	2757.91(3859.07; 33, 494,10000)	764.20(2128.27; 11, 63, 1567)	168.58(869.91; 7, 21, 171)	15.00(101.69; 4, 8, 24)	4.32(2.61; 2, 4, 7)
	$\overline{Q_n}$	2090.56(3498.89; 17, 235,10000)	625.75(1950.56; 7, 42, 1034)	159.54(885.61; 5, 17, 142)	14.80(130.91; 3, 7, 21)	3.82(2.64; 2, 3, 7)
	$\overline{ADM}$	2701.15(3831.98; 31, 470,10000)	772.53(2149.76; 11, 62, 1607)	165.66(859.60; 6, 21, 166)	15.24(117.20; 4, 8, 23)	4.28(2.55; 2, 4, 7)
	$\overline{G}$	2761.00(3855.14; 32, 499,10000)	759.18(2122.92; 11, 63, 1554)	178.68(914.09; 6, 21, 176)	14.87(103.18; 4, 8, 23)	4.32(2.56; 2, 4, 7)
	$\overline{IQR}$	1975.27(3425.78; 15, 201,10000)	600.62(1910.65; 7, 39, 968)	154.97(875.06; 5, 16, 137)	15.11(135.35; 3, 6, 21)	3.73(2.64; 2, 3, 6)
	$\overline{IQR}_{20}$	876.10(2215.30; 10, 85, 2145)	165.84(790.13; 5, 23, 214)	36.50(225.13; 4, 11, 54)	7.64(15.37; 2, 5, 14)	3.14(1.88; 1, 3, 5)
	$\overline{D7}$	1246.79(2531.62; 23, 208, 3930)	188.03(814.39; 9, 37, 265)	39.61(225.36; 6, 16, 62)	8.90(15.15; 3, 7, 16)	3.79(1.95; 2, 3, 6)
	$\overline{ADMs}$	843.66(1976.63; 21, 158, 1964)	116.97(550.13; 9, 32, 174)	27.63(152.96; 5, 14, 47)	8.15(8.06; 3, 7, 15)	3.70(1.84; 2, 3, 6)
	$\overline{ADMi}$	1012.48(2231.48; 19, 169, 2674)	151.28(676.17; 8, 34, 219)	31.55(161.38; 5, 14, 54)	8.40(7.66; 3, 7, 15)	3.66(1.90; 2, 3, 6)
	$\overline{ADMsi}$	884.51(2051.29; 19, 152, 2136)	127.95(568.79; 8, 31, 196)	27.74(125.74; 5, 14, 50)	8.13(8.14; 3, 6, 15)	3.63(1.86; 2, 3, 6)
CS-CUSUM	S_p	3859.86(4345.20; 39, 1066,10000)	1777.06(3312.14; 12, 112,10000)	686.74(2079.51; 7, 31, 1299)	69.49(525.70; 3, 10, 53)	4.98(10.99; 1, 4, 9)
	$\overline{S}$	2532.14(3632.45; 34, 507,10000)	714.18(1981.51; 11, 67, 1496)	166.93(834.57; 6, 23, 185)	15.03(82.64; 3, 8, 24)	3.98(2.88; 1, 3, 7)
	$\overline{Q_n}$	2054.39(3449.31; 17, 241,10000)	617.01(1909.59; 8, 44, 1055)	154.64(846.19; 5, 17, 145)	14.98(117.55; 2, 7, 22)	3.62(2.76; 1, 3, 7)
	$\overline{ADM}$	2479.74(3609.63; 32, 478,10000)	698.01(1950.66; 11, 65, 1452)	163.08(820.76; 6, 22, 181)	14.90(81.96; 3, 8, 24)	3.96(2.98; 1, 3, 7)
	$\overline{G}$	2524.73(3630.27; 34, 501,10000)	720.66(1988.50; 11, 68, 1528)	166.49(826.72; 6, 22, 185)	15.19(87.94; 3, 8, 24)	4.00(2.91; 1, 3, 7)
	$\overline{IQR}$	1950.26(3387.78; 15, 206,10000)	599.72(1892.39; 7, 40, 996)	155.59(860.90; 4, 16, 143)	14.61(104.66; 2, 7, 21)	3.53(2.75; 1, 3, 6)
	$\overline{IQR}_{20}$	859.46(2178.75; 10, 85, 2098)	168.46(792.75; 5, 23, 220)	37.26(242.94; 3, 11, 55)	7.66(19.17; 2, 5, 14)	3.00(1.98; 1, 3, 5)
	$\overline{D7}$	1157.57(2340.42; 24, 219, 3390)	195.76(796.56; 9, 40, 294)	40.75(216.72; 5, 16, 66)	8.90(9.69; 2, 7, 17)	3.46(2.25; 1, 3, 6)
	$\overline{ADMs}$	780.82(1795.05; 22, 168, 1825)	117.56(499.28; 9, 34, 191)	29.16(151.79; 5, 15, 50)	8.11(7.43; 2, 7, 15)	3.33(2.14; 1, 3, 6)
	$\overline{ADMi}$	944.59(2071.07; 20, 176, 2447)	149.56(632.09; 8, 35, 235)	32.09(149.20; 5, 15, 56)	8.28(9.94; 2, 7, 15)	3.33(2.13; 1, 3, 6)
	$\overline{ADMsi}$	877.22(1975.02; 19, 165, 2189)	129.13(544.29; 8, 33, 211)	29.10(114.70; 5, 15, 53)	8.14(11.57; 2, 6, 15)	3.30(2.10; 1, 3, 6)

Table 9: Performance of the EWMA-S chart with the estimator $\overline{ADMsi}$ when k Phase I samples each of size $n = 5$ are used to estimate the unknown parameters.

		ARL(SDRL; 10^{th}, 50^{th}, 90^{th} percentiles) of the unconditional run length distribution				
Scenario	$k(L)$	δ=1.0	δ=1.1	δ=1.2	δ=1.4	δ=1.8
Normality	50(2.185)	370.20(1000.05; 15, 92, 784)	48.72(106.00; 7, 24, 102)	17.39(18.30; 5, 12, 35)	7.23(4.55; 3, 6, 13)	3.54(1.68; 2, 3, 6)
	100(2.385)	370.58(776.53; 23, 139, 846)	49.34(66.35; 9, 29, 106)	18.67(16.12; 6, 14, 36)	7.83(4.48; 3, 7, 13)	3.81(1.74; 2, 3, 6)
	200(2.514)	371.35(613.55; 31, 181, 868)	51.16(55.18; 11, 34, 110)	19.61(15.19; 6, 15, 38)	8.27(4.55; 4, 7, 14)	3.98(1.77; 2, 4, 6)
	300(2.563)	371.01(531.62; 35, 201, 875)	51.68(51.15; 11, 36, 111)	20.05(14.77; 7, 16, 38)	8.43(4.56; 4, 7, 14)	4.06(1.78; 2, 4, 6)
	400(2.586)	369.38(485.75; 37, 211, 865)	52.07(49.71; 12, 37, 110)	20.28(14.75; 7, 16, 39)	8.49(4.55; 4, 7, 14)	4.09(1.78; 2, 4, 6)
	500(2.605)	370.10(462.69; 39, 222, 858)	52.62(49.19; 12, 37, 113)	20.41(14.70; 7, 16, 39)	8.56(4.56; 4, 8, 14)	4.12(1.78; 2, 4, 6)
	1000(2.637)	371.56(414.83; 42, 238, 859)	52.97(47.30; 12, 39, 112)	20.69(14.51; 7, 17, 39)	8.66(4.54; 4, 8, 15)	4.15(1.80; 2, 4, 6)
	3000(2.656)	370.31(377.66; 46, 251, 845)	53.44(45.81; 13, 39, 113)	20.86(14.39; 7, 17, 40)	8.72(4.53; 4, 8, 15)	4.19(1.80; 2, 4, 7)
	∞(2.666)	369.81(361.82; 47, 259, 841)	53.84(45.40; 13, 40, 113)	20.93(14.40; 7, 17, 40)	8.79(4.57; 4, 8, 15)	4.20(1.80; 2, 4, 7)
Diffuse symmetric	50(2.185)	1074.85(2217.19; 23, 209, 3005)	120.85(405.53; 9, 38, 233)	26.98(44.48; 6, 16, 55)	8.61(6.15; 3, 7, 16)	3.87(1.91; 2, 3, 6)
	100(2.385)	1144.70(2071.00; 40, 344, 3061)	104.12(221.56; 12, 48, 227)	27.47(30.16; 7, 19, 56)	9.28(5.84; 4, 8, 16)	4.15(1.93; 2, 4, 7)
	200(2.514)	1180.74(1854.88; 64, 488, 3016)	101.25(143.24; 14, 58, 228)	28.22(25.77; 8, 20, 57)	9.72(5.70; 4, 8, 17)	4.33(1.95; 2, 4, 7)
	300(2.563)	1187.68(1717.32; 78, 561, 2968)	100.58(125.36; 16, 61, 226)	28.56(24.52; 8, 21, 57)	9.92(5.69; 4, 9, 17)	4.42(1.97; 2, 4, 7)
	400(2.586)	1177.26(1606.59; 88, 614, 2878)	99.45(112.62; 16, 64, 222)	28.63(23.72; 8, 22, 57)	9.98(5.65; 4, 9, 17)	4.45(1.98; 2, 4, 7)
	500(2.605)	1179.42(1555.63; 93, 640, 2840)	100.52(108.78; 17, 66, 224)	29.01(23.63; 8, 22, 58)	10.11(5.70; 4, 9, 17)	4.47(1.97; 2, 4, 7)
	1000(2.637)	1162.29(1364.81; 108, 708, 2743)	100.14(100.35; 18, 69, 221)	29.26(22.84; 9, 23, 58)	10.19(5.66; 5, 9, 18)	4.53(1.99; 2, 4, 7)
	3000(2.656)	1138.51(1206.31; 122, 755, 2638)	99.89(94.05; 18, 71, 220)	29.40(22.40; 9, 23, 58)	10.30(5.69; 5, 9, 18)	4.57(2.00; 2, 4, 7)
Localized	50(2.185)	760.48(1804.14; 19, 145, 1762)	85.79(298.28; 8, 30, 164)	22.89(46.86; 5, 14, 46)	8.02(5.62; 3, 7, 14)	3.72(1.81; 2, 3, 6)
	100(2.385)	755.90(1552.70; 31, 228, 1817)	76.17(150.83; 11, 39, 164)	23.19(23.92; 6, 16, 47)	8.62(5.22; 4, 7, 15)	3.99(1.84; 2, 4, 6)
	200(2.514)	728.47(1252.75; 43, 308, 1750)	73.74(95.37; 13, 44, 163)	23.84(20.15; 7, 18, 47)	9.08(5.20; 4, 8, 16)	4.18(1.88; 2, 4, 7)
	300(2.563)	723.41(1111.22; 52, 353, 1736)	73.42(83.45; 13, 47, 162)	24.28(19.66; 7, 19, 48)	9.23(5.20; 4, 8, 16)	4.25(1.89; 2, 4, 7)
	400(2.586)	707.54(1013.18; 58, 374, 1679)	74.15(79.58; 14, 49, 163)	24.37(19.16; 8, 19, 48)	9.26(5.12; 4, 8, 16)	4.27(1.89; 2, 4, 7)
	500(2.605)	704.89(946.29; 62, 391, 1674)	74.55(76.72; 14, 50, 162)	24.55(19.02; 8, 19, 48)	9.38(5.19; 4, 8, 16)	4.32(1.89; 2, 4, 7)
	1000(2.637)	690.62(799.42; 69, 431, 1624)	74.21(70.77; 15, 52, 162)	24.85(18.57; 8, 20, 48)	9.49(5.18; 4, 8, 16)	4.36(1.90; 2, 4, 7)
	3000(2.656)	672.24(706.00; 75, 451, 1544)	74.17(67.22; 15, 54, 160)	25.01(18.20; 8, 20, 48)	9.53(5.12; 4, 8, 16)	4.38(1.89; 2, 4, 7)

Diffuse symmetric denotes diffuse symmetric variance contaminated normal environment; Localized denotes localized variance contaminated normal environment.

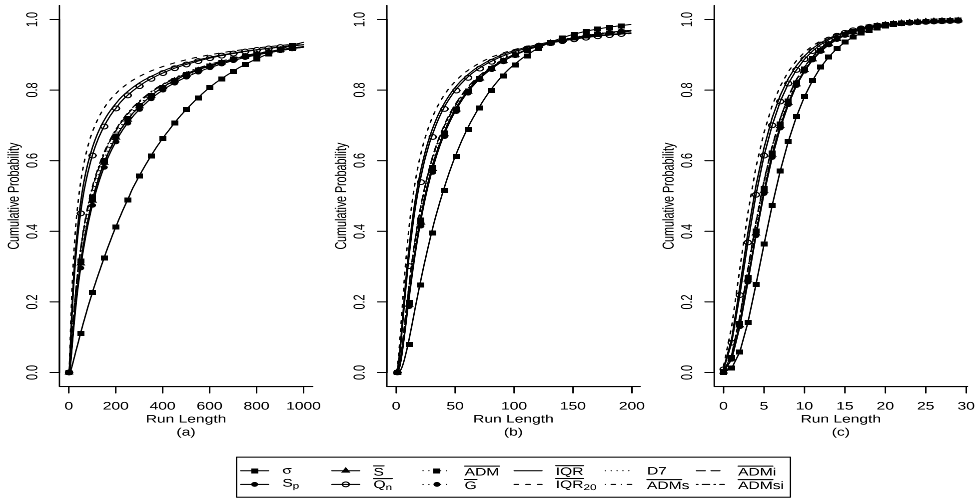

Figure 1: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ and ARL_0=370 under uncontaminated normal environment: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

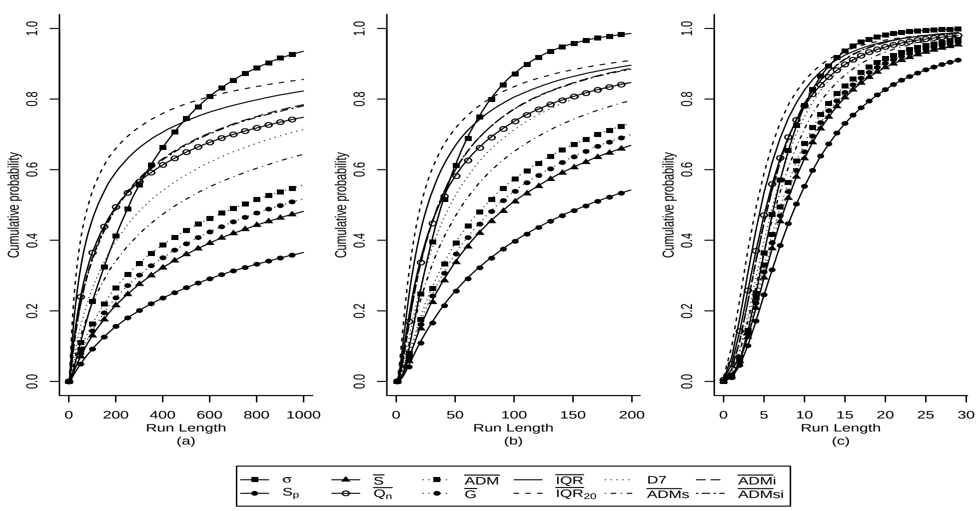

Figure 2: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ under diffuse symmetric variance contaminated normal environment: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

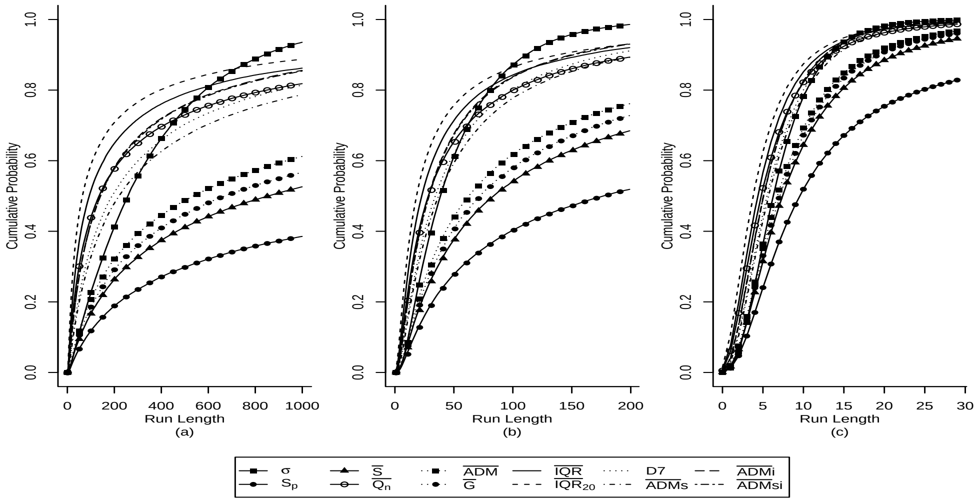

Figure 3: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ under diffuse asymmetric variance contaminated normal environment: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

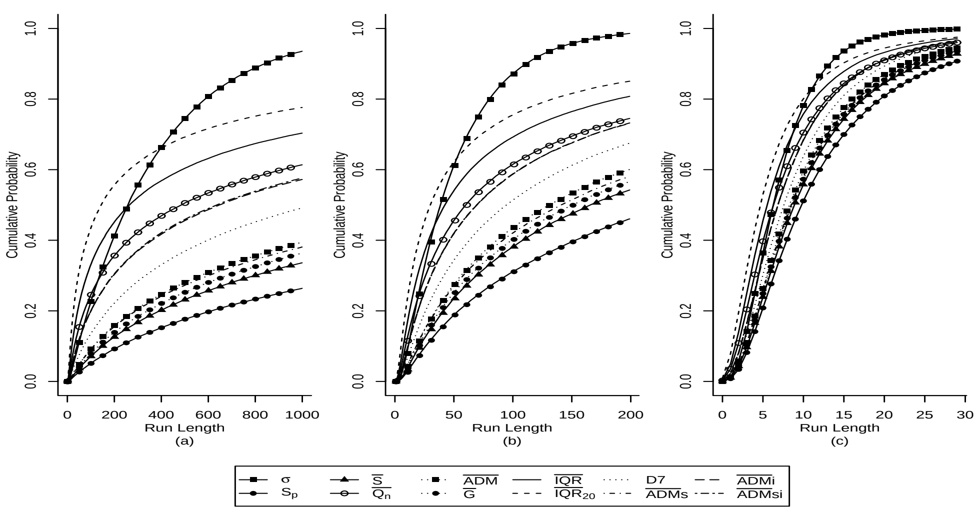

Figure 4: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ under diffuse mean contaminated normal environment: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

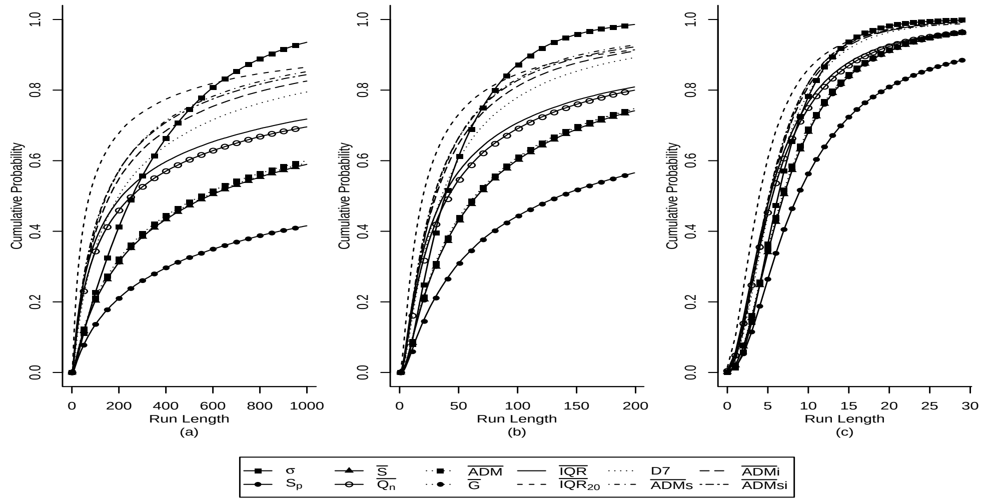

Figure 5: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ under localized variance contaminated normal environment: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

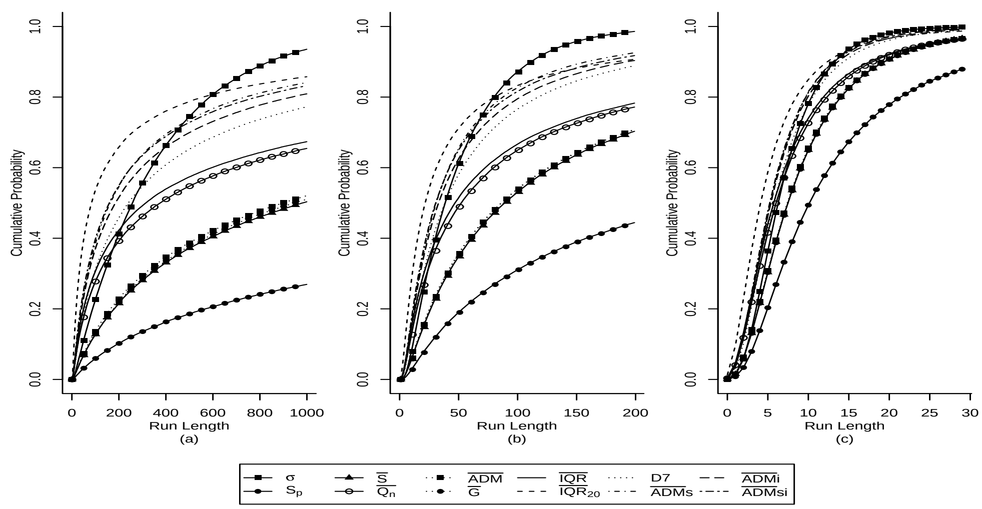

Figure 6: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ when a single step shift in the variance is present in Phase I: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

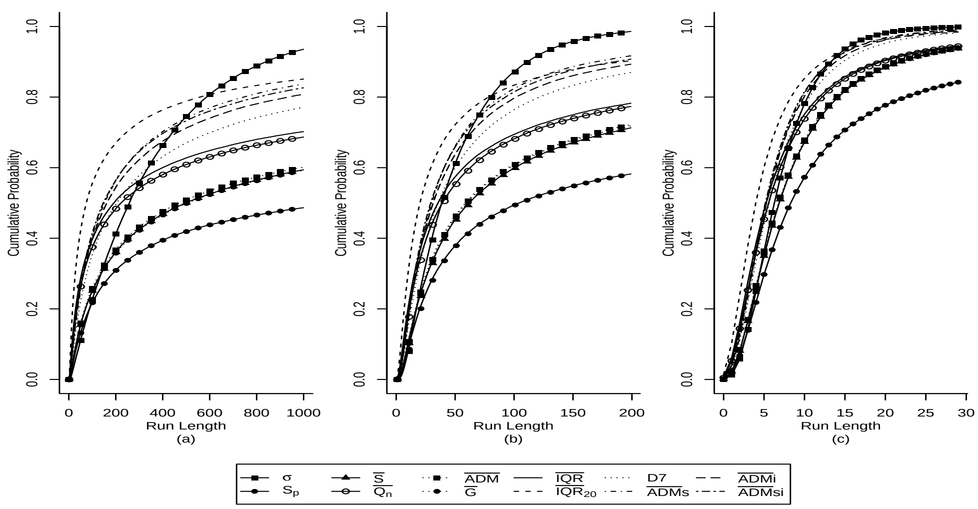

Figure 7: The IC and OC CDF curves for the EWMA-S chart with different estimators and with known parameters for $k = 50, n = 5$ when multiple step shifts in the variance are present in Phase I: (a) IC CDF curves; (b) OC CDF curves when $\delta = 1.1$; (c) OC CDF curves when $\delta = 1.4$.

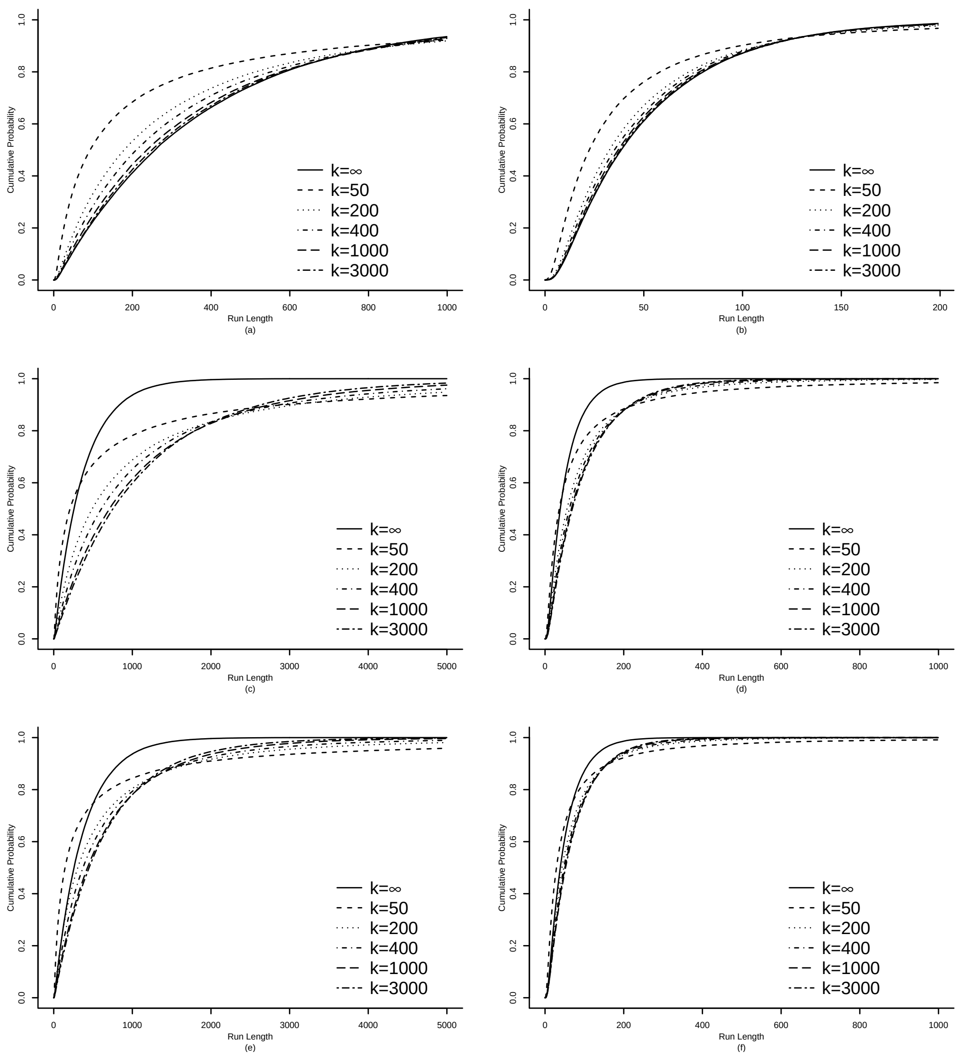

Figure 8: The IC and OC CDF curves for the EWMA-S chart with the estimator $\overline{ADM}si$ for $n = 5$ and $k \in \{50, 200, 400, 1000, 3000, \infty\}$ under various scenarios: (a) IC CDF curves under uncontaminated normal environment; (b) OC CDF curves when $\delta = 1.1$ under uncontaminated normal environment; (c) IC CDF curves under diffuse symmetric variance contaminated normal environment; (d) OC CDF curves when $\delta = 1.1$ under diffuse symmetric variance contaminated normal environment; (e) IC CDF curves under localized variance contaminated normal environment; (f) OC CDF curves when $\delta = 1.1$ under localized variance contaminated normal environment.