

Self-Starting Control Chart for Simultaneously Monitoring Process Mean and Variance

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A self-starting control chart, based on the likelihood ratio test (LRT) and the exponentially weighted moving average (EWMA) procedure, is proposed for monitoring the process mean and variance simultaneously when the process parameters are unknown. A table is presented to assist in the design of the control chart with different parameters. Its in-control (IC) average run length (ARL) can be evaluated by a two-dimensional Markov chain model. Moreover, the diagnostic aids of the proposed chart are given. Monte Carlo simulation results compared with some competing methods in literature show that the proposed approach has quite satisfactory charting performance across a range of possible shifts when the process parameters are unknown, even including the detection of the decrease in variability. A real data example from industrial manufacturing is used for demonstrating its implementation.

Keywords: Average Run Length; Control Charts; Diagnostics; Quality Engineering; SPC.

1. Introduction

Since Shewhart first developed the statistical process control (SPC) chart concept, control charts have been an essential statistical tool in industry. It is desirable to construct a control chart that can not only detect changes in the process mean, but also is sensitive to the shifts in the process variability. In general, there are two main methods of developing control schemes for jointly monitoring the mean and variance of the process. One is to combine two control charts that are designed for monitoring the mean and variability respectively, such as the combination of Shewhart \bar{X} (or X) and R (MR or S) charts (Saniga 1989, Rahim 1989, Costa 1993, Rahim and Costa 2000) and the combination of EWMA-type charts (Domangue and Patch 1991, Reynolds and Stumbos 2001). The other one is to use omnibus-type test statistics to implement a single control scheme for detecting shifts in both mean and variance (Domangue and Patch 1991, Chen *et al.* 2001, Wu and Yu 2005, Zhang *et al.* 2008).

All of the methods mentioned above assume that the parameters of the process are known. In most industrial applications, however, the process parameters to be monitored are unknown and have to be estimated in a special Phase I calibration exercise. Some authors have recommended using 20-30 samples with four to five observations each to estimate the process parameters for traditional control charts (Ryan 2000, Montgomery 2005). Quesenberry (1993) and Jones *et al.* (2001, 2004) have investigated the effect of the estimated parameters on the performance of

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traditional control charts. A recent literature review paper by Jensen *et al.* (2006) provides a thorough discussion of the effects of parameters estimation on control chart performance. They conclude that, when the number of reference samples is small, control charts with estimated parameters produce a large bias in the in-control (IC) ARL and reduce the sensitivity of the chart in detecting process changes as measured by the out-of-control (OC) ARL. Moreover, after short runs, false alarm probabilities from the charts increase drastically (Bischak and Trietsch 2007). A direct solution to this problem is to increase the Phase I sample size to reduce the variability of the estimates, especially the process variance. In most cases, however, it may not be possible to wait for the accumulation of sufficiently large subgroups because the users usually want to monitor the process at the start-up stages, such as job-shop environment. A different remedy involves using some self-starting methods, which have been developed accordingly that update the parameter estimators along with new observations and simultaneously monitor the process to see whether the process is in control or not (Hawkins 1987, Hawkins and Olwell 1998, Quesenberry 1991, 1995, Sullivan and Jones 2002). To enhance the performance of the method of Quesenberry (1995), Li and Wang (2008) propose an adaptive Cusum of Q chart, which can detect a range of shifts adaptively. In particular, Hawkins *et al.* (2003), Hawkins and Zamba (2005a,b) propose a change-point model based on the likelihood ratio for on-line monitoring that can also be seen as self-starting methods.

In many applications, as mentioned above, the parameters of the process are always not known before running a chart. Therefore, it is desirable to find a control chart that performs well even when the parameters of the process are unknown. The main objective of this paper is to develop a procedure that will monitor the changes of process shifts in both the mean and the variance without collecting a sufficient large sample of IC observations. So a self-starting control chart based on the LRT and the EWMA procedure (SSELR) is proposed for monitoring the process mean and variance simultaneously when the process parameters are unknown. Without these IC observations, we do not have information of the process parameters, so our SSELR chart is a self-starting control chart when the IC process parameters μ and σ are unknown. This means that it is not necessary to assemble a large number of reference samples before the control chart begins, although it is advisable to collect a few preliminary samples. So it has the advantage that the process parameters do not have to be known, which may make it useful for short production run situations. Because our proposed SSELR chart is a single chart, the design and operation of the monitoring scheme can be greatly simplified compared with the combination-type charts. The proposed SSELR chart has the following positive features: 1) the analysis approach is more reliable than Monte Carlo simulation because its IC ARL can be evaluated by a two-dimensional Markov chain model; 2) due to fully inheriting the advantages of classical LRT, it is quite robust and sensitive to various types of shifts; 3) it can effectively detect the decrease in variability; 4) it is able to handle the case that the sample size is one; 5) it can diagnose when the process has gone out of control and which parameter or parameters have underwent a shift.

The remainder of this paper is organized as follows. In the next section, our proposed control chart and the diagnostic aids are presented. The numerical comparisons with other competing procedures are shown in Section 3. The implementation of our proposed method is illustrated in Section 4 by a real data example from industrial manufacturing. Several remarks conclude this paper in Section 5. The detailed derivations of the testing statistics are deferred to the Appendix.

2. SSELR chart for monitoring process mean and variance

2.1 The method of the SSELR chart

Let $\mathbf{x}_t = (x_{t1}, \dots, x_{tn}), t = 1, 2, \dots$ denote a sequence of samples of size $n (\geq 1)$ taken on a quality characteristic X . It is assumed that, for each t, x_{t1}, \dots, x_{tn} are identically and independently distributed (i.i.d) random variables from a normal $N(\mu, \sigma^2)$ distribution. Departures from control may take the form of shifts in μ and/or σ . These shifts will be thought of as starting at some instant τ , and persisting for some time. The traditional model is given by

$$\mathbf{x}_t \sim \begin{cases} N(\mu, \sigma^2), & \text{if } t = 1, 2, \dots, \tau, \\ N(\mu + \delta\sigma, (\gamma\sigma)^2), & \text{if } t = \tau + 1, \dots \end{cases} \quad (1)$$

We assume that none of the parameters $\mu, \sigma, \delta, \gamma$ and τ are known a priori. If a shift occurs, i.e. $\delta \neq 0$ and(or) $\gamma \neq 1$, it is important to signal its presence as soon as possible after it takes place, without undue false alarm.

Write $\mu_t = \frac{1}{nt} \sum_{i=1}^t \sum_{j=1}^n x_{ij}$ for the mean of the first t process readings and $W_t = \sum_{i=1}^t \sum_{j=1}^n (x_{ij} - \mu_t)^2$ for the sum of squared deviations of the first t readings from their mean. The sample variance of the first t readings is then given by $s_t^2 = \frac{W_t}{tn-1}$. If the process is in control, then $\mu_t \sim N(\mu, \frac{\sigma^2}{nt})$ and $\frac{W_t}{\sigma^2} \sim \chi^2(nt - 1)$. The calculation of the running mean and variance are considerably simplified by the fact that they can be written using the updating formula as follows:

$$\mu_t = \mu_{t-1} + \frac{\bar{x}_t - \mu_{t-1}}{t}, \quad (2)$$

and

$$W_t = W_{t-1} + n(t - 1)(\mu_t - \mu_{t-1})^2 + \sum_{j=1}^n (x_{tj} - \mu_t)^2, \quad (3)$$

where $\bar{x}_t = \frac{1}{n} \sum_{j=1}^n x_{tj}$ (The derivations of Equations (2) and (3) are presented in the Appendix). Standardizing each reading using the running mean and standard deviation of the preceding observations gives:

$$T_{tj} = \frac{x_{tj} - \mu_{t-1}}{s_{t-1}}, j = 1, 2, 3, \dots, n.$$

We know that

$$x_{tj} - \mu_{t-1} \sim N\left(0, \frac{(nt - n + 1)\sigma^2}{nt - n}\right),$$

$$\frac{(nt - n - 1)s_{t-1}^2}{\sigma^2} \sim \chi^2(nt - n - 1)$$

Under the normal distribution assumption, T_{tj} follows a scaled Student's t dis-

tribution:

$$\sqrt{\frac{nt-n}{nt-n+1}} T_{tj} \sim t_{nt-n-1}, j = 1, 2, 3, \dots, n.$$

Define $a_t = \sqrt{\frac{nt-n}{nt-n+1}}$. The exact cumulative distribution function (CDF) of T_{tj} is then given by

$$Pr[T_{tj} < T] = T_{nt-n-1}(a_t T),$$

where $T_{nt-n-1}(\cdot)$ stands for the CDF of the Student's t distribution with $nt-n-1$ degrees of freedom. Then the transformation

$$w_{tj} = \Phi^{-1}[T_{nt-n-1}(a_t T_{tj})]$$

will transform the quantity T_{tj} into a random variable w_{tj} that has an exact $N(0, 1)$ distribution for all $t \geq 3$ when the process is in control. When an assignable cause occurs after some subgroups, say, τ subgroups, the distribution of $w_{tj}, t = \tau + 1, \tau + 2, \dots, j = 1, 2, \dots, n$, is different from that of $w_{tj}, t = 1, 2, \dots, \tau, j = 1, 2, \dots, n$. The difference between them will be used in our method to detect the assignable cause.

Zhang *et al.* (2008) propose an ELR chart based on the LRT when the parameters are exactly known a priori. We, however, develop our new chart on the condition that the process parameters are unknown.

Given a sample $\mathbf{x}_t, t \geq 3$, based on the quantity w_{tj} , consider the following hypothesis test

$$H_0 : \delta = 0 \text{ and } \gamma = 1 \longleftrightarrow H_1 : \delta \neq 0 \text{ or } \gamma \neq 1.$$

It is straightforward to obtain the generalized likelihood ratio statistic as follows

$$l_t = n(\bar{w}_t^2 + S_t^2 - \ln S_t^2 - 1), \tag{4}$$

where $\bar{w}_t = \frac{1}{n} \sum_{j=1}^n w_{tj}$ and $S_t^2 = \frac{1}{n} \sum_{j=1}^n (w_{tj} - \bar{w}_t)^2$. It can be easily checked that

$l_t \xrightarrow{\mathcal{L}} \chi^2(2)$ as $t \rightarrow \infty$ (The derivation of Equation (4) and the asymptotic distribution are presented in the Appendix). Obviously, a large l_t leads to rejecting the null hypothesis. The terms \bar{w}_t^2 and $S_t^2 - \ln S_t^2$ contribute to the changes of the process mean and variance, respectively. Note that the function $z - \ln z$ is monotonically increase (decrease) when $z > 1$ ($0 < z < 1$) and attains its minimum at $z = 1$. Hence, the testing statistics l_t will be sensitive to both the increase and decrease in variance. Unlike some other test statistics in the literature, l_t is a likelihood ratio derived under the setting in which the process mean and variance may change, and thus naturally adapts to be sensitive to various types of shift combinations. The comparative study in the next section also verifies this point.

In order to detect small or moderate shifts effectively, we incorporate EWMA procedure to the construction of l_t . Here the EWMA scheme is not to directly average the l_t statistics but rather to get more precise "estimates" of the current process mean and variance. To be specific, two EWMA statistics based on the sample mean \bar{x}_t and sample variance S_t^2 are given by

$$u_t = \lambda \bar{w}_t + (1 - \lambda)u_{t-1}, \tag{5}$$

$$v_t = \lambda S_t^{*2} + (1 - \lambda)v_{t-1}, \tag{6}$$

where $S_t^{*2} = \sum_{j=1}^n (w_{tj} - u_t)^2/n$, $u_0 = 0$, $v_0 = 1$, and λ is the smoothing parameter satisfying $0 < \lambda < 1$. In general, a smaller λ leads to a quicker detection of smaller shifts (Lucas and Saccucci 1990).

Note that the statistic v_t in (6) is different from its counterpart used in Chen *et al.* (2001). Here, the moving average estimation of process mean u_t is used in the variance estimation to replace \bar{w}_t . It would be expected to be more accurate by using this sequentially updated estimation and thus may improve the ability to detect the possible process change. Our extensive simulation results demonstrate this point. In fact, this formulation is essentially equivalent to its multivariate counterpart used in Huang *et al.* (2007).

Finally, substituting u_t and v_t for \bar{w}_t and S_t^2 in (4), we can obtain the self-starting EWMA LRT (SSELR) charting statistics

$$SSELR_t^* = n(u_t^2 + v_t - \ln(v_t) - 1), \quad t = 1, 2, \dots \tag{7}$$

In practice, we can omit the constants in (7) and plot

$$SSELR_t = u_t^2 + v_t - \ln(v_t), \quad t = 1, 2, \dots \tag{8}$$

A signal is addressed when $SSELR_t$ exceeds a given upper control limit h , which is chosen to obtain the given specified IC ARL. For some given IC ARL and sample size $n = 1(1)15$, the control limits for each chart are tabulated in Table 1. A Fortran program that easily finds the control limits for each chart given a desired overall IC ARL, a smoothing constant λ , is available from the authors. The smoothing constant λ in Equation (5) and (6) are chosen to be equal to simplify the computation although we can certainly use different smoothing constants that lead to quicker detection of different shifts (Lucas and Saccucci 1990).

Insert Table 1 about here.

Since the w_{tj} statistics are exactly i.i.d Normal random variables under IC condition, that is, the IC mean and variance of the process are both known, the two-dimensional Markov chain model developed in Zhang *et al.* (2008) can be used to evaluate the IC ARL of the SSELR chart.

Our SSELR is not the first self-starting proposal. However, there are two major differences between our proposed scheme and Hawkins (1987): First, our considered type of data set is rational subgroup of size $n \geq 1$, while only the individual observations are investigated by Hawkins (1987). Another difference is that two pairs of Cusums are set up in Hawkins (1987): one testing for constancy of location of the process, and the other for constancy of the variance. However, we propose a scheme to monitor the process mean and variance simultaneously by using one single chart based on the LRT. It is shown that the procedure performs well in detecting changes in the process. The differences between our SSELR chart and Hawkins and Zamba (2005b) are shown in the comparative study in the next section.

2.2 The diagnostic aids of the SSELR chart

In practice, when a special cause produces a change in one or more process parameters, it is important to detect this change quickly, and it is also significant to diagnose when the process has gone out of control and which parameter or parameters have underwent a shift after an OC signal is given. Such a diagnostic aid

is particularly important in our SSELR chart, which is a single control chart for monitoring both process mean and variance shifts simultaneously. The diagnostic aids to locate the change point in the process and to isolate the type of parameter change will help a practitioner to identify and eliminate the special cause of a problem quickly and easily. Following the idea of Zou *et al.* (2007), we propose a diagnostic method based on the maximum likelihood estimator of the change point τ to assist in the diagnosis of our SSELR chart. In this section, the statistics of \bar{w}_t are used in our diagnostic aids of the SSELR chart.

Let $\{\bar{w}_j, j = 1, 2, \dots, k\}$ denote all the historical and collected future samples. Then the logarithm of the likelihood function is given by

$$-\frac{1}{2} \sum_{j=1}^k (\log(2\pi\sigma_j^2) + \frac{(\bar{w}_j - \mu_j)^2}{\sigma_j^2}).$$

If the data are collected under IC conditions, the maximum value of the logarithm of likelihood function is

$$l_0 = -\frac{k}{2} \log(2\pi) - \frac{k}{2} \log \hat{\sigma}_k^2 - \frac{k}{2}$$

where

$$\hat{\sigma}_k^2 = \frac{\sum_{j=1}^k (\bar{w}_j - \bar{\bar{w}}_k)^2}{k}, \bar{\bar{w}}_k = \frac{\sum_{j=1}^k \bar{w}_j}{k}.$$

When there is a step shift after the k_1^{th} sample, the corresponding maximum value is

$$l_1 = -\frac{k}{2} \log(2\pi) - \frac{k_1}{2} \log \hat{\sigma}_{1,k_1}^2 - \frac{k_2}{2} \log \hat{\sigma}_{2,k_1}^2 - \frac{k}{2}$$

where

$$\hat{\sigma}_{1,k_1}^2 = \frac{\sum_{j=1}^{k_1} (\bar{w}_j - \bar{\bar{w}}_{1,k_1})^2}{k_1}, \hat{\sigma}_{2,k_1}^2 = \frac{\sum_{j=k_1+1}^k (\bar{w}_j - \bar{\bar{w}}_{2,k_1})^2}{k_2},$$

$$\bar{\bar{w}}_{1,k_1} = \frac{\sum_{j=1}^{k_1} \bar{w}_j}{k_1}, \bar{\bar{w}}_{2,k_1} = \frac{\sum_{j=k_1+1}^k \bar{w}_j}{k_2}, k_2 = k - k_1.$$

Thus, the classical likelihood ratio statistic is defined by

$$lr(k_1, k) = -2(l_0 - l_1) = k \log(\hat{\sigma}_k^2 (\hat{\sigma}_{1,k_1}^2)^{-k_1/k} (\hat{\sigma}_{2,k_1}^2)^{-k_2/k}).$$

The estimator of the change point, τ , of a step shift in parameter(s) including process mean and(or) variance is given by

$$\hat{\tau} = \arg \max_{2 \leq k_1 \leq k-2} \{lr(k_1, k)\}.$$

It is necessary to justify which parameter or parameters have shifted after a signal occurs. After obtaining the change point estimator, $\hat{\tau}$, we may consider the parametric test method as follows:

- The t -test for the mean change using degrees of freedom $k - 2$ and test statistic

$$t_{\mu} = \frac{\sqrt{\hat{\tau}(k - \hat{\tau})/k}(\bar{w}_{1,\hat{\tau}} - \bar{w}_{2,\hat{\tau}})}{\sqrt{(\hat{\tau}\hat{\sigma}_{1,\hat{\tau}}^2 + (k - \hat{\tau})\hat{\sigma}_{2,\hat{\tau}}^2)/(k - 2)}}$$

- The F -test for the variance change using degrees of freedom $\hat{\tau} - 1$ and $k - \hat{\tau} - 1$. The test statistic is

$$F_{\sigma} = \frac{\hat{\tau}(k - \hat{\tau} - 1)\hat{\sigma}_{1,\hat{\tau}}^2}{(\hat{\tau} - 1)(k - \hat{\tau})\hat{\sigma}_{2,\hat{\tau}}^2}.$$

3. Performance and Comparisons

In this section, the comparisons among our proposed SSELR chart and the MEW chart of Chen *et al.* (2001) (denoted as MEW) and ELR chart of Zhang *et al.* (2008) (denoted as ELR) for rational subgroups $n = 5$ and the chart of Hawkins and Zamba (2005b) (denoted as CP) based on the change point model for individual observations are carried out by Monte Carlo simulation based on 10,000 replications, such that the standard errors of the estimates are less than 2%, enabling us to draw reasonable conclusions. Note that the method of Chen *et al.* (2001) and Zhang *et al.* (2008) rely on the availability of the values of the IC parameters, so they are not standard alternatives. It is, however, still interesting to show the difference of our SSELR chart with IC process parameters unknown. Because it is known that the performance of self-starting method is affected by the change point τ , we assesses the OC ARL performance of our chart for different values of τ of IC samples before a shift occurs.

3.1 Comparison with MEW and ELR

For simplicity, we only consider the case of overall IC ARL=370 when $n = 5$, $\lambda = 0.2$. The control limit is set to be 1.2456 from Table 1. Although the IC ARL of our SSELR can be evaluated by the Markov chain procedure, in this paper, the performance when the process is out of control is studied by simulating sequence of standard normal data. Random samples from a fixed Normal distribution are generated for $\tau = 10, 25, 50, 100, 250$ and 500 , after which shifts are in either the mean or variance or are simultaneous. The process mean and process standard deviation are changed from 0 to δ and from 1 to γ , respectively. The results are listed in Table 2.

Insert Table 2 about here.

Table 2 shows that our proposed SSELR chart performs almost equally well for all values of τ when detecting a large shift. Naturally, the OC ARL will be affected by the number of reference samples gathered before a shift actually occurs. The benefit is much more obvious in the case of detecting a small to moderate shift than in detecting a large shift. Because the SSELR chart updates the parameter estimation with new samples, the more IC future samples one collects, the more sensitive the SSELR chart is to a small to moderate shift. Moreover, it can be seen that the performance of our SSELR with about 25 start-up observations is good enough, which implies we do not need to gain much more satisfactory performance at the cost of collecting many more observations. Our SSELR chart has comparable

performance with the MEW chart and ELR chart, although they are constructed on the assumption that the IC process parameters are known.

As an anonymous reviewer pointed out, it is interesting to show the optimal parameters of λ and h for detecting different combinations of shifts (δ, γ) . Table 3 presents the (λ^*, h^*, ARL^*) by minimizing the OC ARL under the constraint IC ARL=370.4 and shift position $\tau = 10$, where λ^* and h^* are the optimal parameters of λ and h and ARL^* is the minimum ARL. We find the minimum OC ARL by the standard grid search algorithm, i.e., we compute the OC ARL with $\lambda = 0.02(0.02)1.00$ and find the minimum OC ARL for different combinations of shifts (δ, γ) . Although Table 3 only lists the cases $n=3$ and $n=7$, other cases show similar patterns, thus, omitted here.

Insert Table 3 about here.

From Table 3, we can draw some general conclusions. First, for fixed δ , the optimal λ^* is large for large deviations of γ , i.e., too small a γ (e.g. $\gamma = 0.2$) or too large a γ (e.g. $\gamma = 1.8$). However, this is not always true. For example, when $\delta \geq 1.5$ and $\gamma > 1.0$, the optimal λ^* is not larger than that when $\delta \geq 1.5$ and $\gamma = 1.0$. This is not expected and warrants further investigation. Similar conclusions can be drawn for fixed γ . Second, the optimal ARL^* when $n = 7$ is greatly reduced relative to that when $n = 3$. For example, when $(\delta, \gamma) = (0, 1.4)$ and $(\delta, \gamma) = (0.5, 1.0)$, the optimal ARL^* are 10.705 and 11.622 when $n = 7$ while the optimal ARL^* are 175.840 and 124.429 when $n = 3$. This implies the performance of our SSELR can be greatly improved if we can collect data with large sample size.

3.2 Comparison with CP

The change point approach is proposed by Hawkins and Zamba (2005b). The simulation results show this approach is always nearly the best for detecting the mean shifts and(or) variance shifts. Thus, it makes an appropriate benchmark against which to compare the SSELR chart.

The SSELR chart and the CP of Hawkins and Zamba (2005b) are compared when the IC ARL are made to be about 500, such that they are comparable in the IC condition. When $n = 1$, $\lambda = 0.2$, the control limit is 2.2187 from Table 1. Similarly, we consider different τ . Note that the bolded values in Table 4 are smaller.

Insert Table 4 about here.

From Table 4, we can draw some conclusions as follows:

- When $\tau + 1=50$, the SSELR chart has worse performance than the CP approach with mean shift $\delta < 1$. When the mean shift $\delta > 1$, the SSELR chart has better performance than the CP approach.
- When $\tau + 1=10$, the SSELR chart always performs better than CP with mean shift $\delta \leq 1.5$, despite the fact that it has a little worse performance when $\delta = 2.0$. However, the performance gain of our SSELR to CP outweighs the performance loss.
- When $\tau + 1=250$, the SSELR chart has better performance for most of the cases considered here.
- When both δ and γ are moderate to large ($\delta \geq 1.0$ and $\gamma \geq 1.25$), SSELR performs better than CP.
- It is not necessary to collect more IC samples with more cost to have better performance.

Although our proposed SSELR chart does not always perform better than the CP approach, it has its own advantages. First, it is easy to design in practice. All a

practitioner needs to do is specify a combination of IC ARL, smoothing parameter λ , sample size n and control limit h , which can be found in Table 1. A Fortran program is available from the authors upon request. Moreover, the IC ARL of SSELR chart can be obtained through a two-dimensional Markov chain model. Once these parameters are chosen, they are not changed during the monitoring process. This is not the case for the CP approach, in which the critical value h_n has to be changed as each new observation enters the monitoring process. Second, it is easy to compute the monitoring statistics for SSELR chart. For the change point approach, because it is needed to calculate the two sample statistic $G_{k,n}$ for every possible split point $1 \leq k < n$ to find the $G_{max,n}$, the ever-growing storage requirement for the two samples seems less convenient.

4. A Real Data Example

In this section, our proposed chart will be illustrated by an example of two laboratories carrying out routine indirect(instrumental) assays for precious metals of batches of a feedstock, which was used in Hawkins (1987). Interested practitioners in the example and the data from the two laboratories are referred to Table 3 in Hawkins (1987) for more details, so the data sets are not presented here.

Although in the previous section, the comparisons among our proposed SSELR chart and some competing methods are carried out, we incorporate some results in Hawkins (1987) to better illustrate the efficiency of our method. As discussed before, the parameters of our proposed SSELR chart are $n = 1, \lambda = 0.2$. For this SSELR chart the control limit is $h = 1.8818$ and the IC ARL is about 100, which is the same as the self-starting Cusum chart employed by Hawkins (1987), in which the control limit is $h = 6$ and the reference value is $k = 0.25$.

The results of our method are tabulated in Table 5 and Table 6 for Laboratory 1 data and Laboratory 2 data, respectively. Note that the columns labelled " P_t " and " P_F " are the P-values of the statistics " t_μ " and " F_σ ". To give a clear picture of the performance our SSELR chart, the SSELR readings and control limit $h = 1.8818$ are further shown in Figure 1.

Insert Figure 1 about here.

Insert Table 5 and Table 6 about here.

From Table 5, we can see that the SSELR chart signals a shift at the 30th sample (i.e., $t=30$) for Laboratory 1. Because two data samples are required to get the estimators of initial mean and standard deviation to get the first transformed statistic w_t , and two readings of w_t are required to get the standard deviation of the transformation data, we can diagnose from the fourth data with our proposed diagnostic aids. Then by looking at the values of $lr(k_t, k)$ for Laboratory 1 ($t = 4, 5, \dots, 28$) in Table 5, we can find that its maximum occurs at $t = 15$ with $lr(15, 30) = 9.8594$. Note from model (1) the process undergoes a shift after τ samples. This maximum indicates the same change-point location, τ , of the shift with Hawkins (1987). From the P-values " $P_t = .001598$ " and " $P_F = .715973$ ", we can conclude the process mean has underwent a significant shift if we choose a significant level α , say, 0.05 or even less. Moreover, by computing the test statistics, t_μ and F_σ , we obtain $t_\mu = -3.2481$ and $F_\sigma = 1.3703$. For the significant level, $\alpha = 0.05$, it follows that $|t_\mu| = 3.2481 > 2.0484 = |T(0.025; 28)|$, and $F_\sigma = 1.3703 < 2.4244 = F(0.95; 14; 15)$, where $T(0.025; 28)$ and $F(0.95; 14; 15)$ are the lower percentiles of the Student-t distribution with 28 degrees of freedom and of the F-distribution with 14 and 15 degrees of freedom. Hence, our SSELR indicates the same time when there is a shift and the same parameter which undergoes a shift with Hawkins (1987). Similar analysis is conducted for Laboratory 2, and

the results are shown in Table 6. The SSELR chart signals a shift at the 29th sample (i.e., $t=29$), which is 2 samples earlier than Hawkins (1987). The maximum occurs at $t = 22$ with $lr(22, 29) = 9.2102$ and $t\mu = -2.9803$, and $F_\sigma = 0.4973$. The P-values " $P_t = .003165$ " and " $P_F = .114379$ " strongly indicate the process mean has a significant shift. For $\alpha = 0.05$, it follows that $|t\mu| = 2.9803 > 2.0518 = |T(0.025; 27)|$, and $F_\sigma = 0.4973 < 3.8649 = F(0.95; 21; 6)$. Hence, our SSELR concludes that there is a shift in the process mean after sample 22, which is the same with Hawkins (1987), but signals 2 samples earlier.

5. Conclusions and Considerations

Based on the LRT and the EWMA procedure, we proposed an SSELR chart to detect the process mean and variance simultaneously when the process parameters are unknown. The IC ARL of our SSELR chart can be analyzed through a two dimensional Markov chain method and it performs well in the case when process parameters are unknown but some historical samples are available. We also give a useful tool based on the maximum likelihood ratio to diagnose the position of shift. In practical applications, if one wants to get information about which parameter or parameters have been changed, two parameter tests can then be applied to aid the proposed chart.

Usually a decrease in the variance corresponds to an improvement in the measurement process as long as other parameters do not change. The simulation results show that our new chart can detect various types of shift in the process including the decrease of the variance. Considering the easy computation and powerful performance of our SSELR chart, we believe it is useful for practitioners in practice.

In this paper, the univariate data samples of sample size $n \geq 1$ are considered. Sullivan and Jones (2002) propose a self-starting control chart for multivariate individual observations. Due to the complexity of the multivariate data, generalizing the idea of this paper to multivariate case is still challenging. This is under research of the authors.

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Appendix

The derivation of Equation (2):

$$\begin{aligned} \mu_t &= \frac{1}{tn} \sum_{i=1}^t \sum_{j=1}^n x_{ij} \\ &= \frac{1}{tn} \left(\sum_{i=1}^{t-1} \sum_{j=1}^n x_{ij} + \sum_{j=1}^n x_{tj} \right) \\ &= \frac{1}{tn} \left((t-1)n\mu_{t-1} + n\bar{x}_t \right) \\ &= \mu_{t-1} + \frac{\bar{x}_t - \mu_{t-1}}{t} \end{aligned}$$

The derivation of Equation (3):

$$\begin{aligned} W_t &= \sum_{i=1}^t \sum_{j=1}^n (x_{ij} - \mu_t)^2 \\ &= \sum_{i=1}^{t-1} \sum_{j=1}^n (x_{ij} - \mu_t)^2 + \sum_{j=1}^n (x_{tj} - \mu_t)^2 \\ &= \sum_{i=1}^{t-1} \sum_{j=1}^n (x_{ij} - \mu_{t-1} + \mu_{t-1} - \mu_t)^2 + \sum_{j=1}^n (x_{tj} - \mu_t)^2 \\ &= \sum_{i=1}^{t-1} \sum_{j=1}^n (x_{ij} - \mu_{t-1})^2 + n(t-1)(\mu_t - \mu_{t-1})^2 + \sum_{j=1}^n (x_{tj} - \mu_t)^2 \\ &= W_{t-1} + n(t-1)(\mu_t - \mu_{t-1})^2 + \sum_{j=1}^n (x_{tj} - \mu_t)^2 \end{aligned}$$

The derivation of Equation (4):

As

$$w_{tj} \sim N(0, 1), j = 1, 2, \dots, n,$$

then the Log likelihood function under H_0 is

$$l_0 = -\frac{n}{2} \log 2\pi - \frac{\sum_{j=1}^n w_{tj}^2}{2}$$

When we get n samples, the MLE of the parameters are

$$\hat{\mu} = \bar{w}_t, \quad \hat{\sigma}_2 = S_t^2,$$

respectively, then the Log likelihood function under H_1 is

$$l_1 = -\frac{n}{2} \log 2\pi\hat{\sigma}_t^2 - \frac{n}{2}$$

Then we have the LR_t statistics as follows:

$$-2(l_0 - l_1) = n(\bar{w}_t^2 + S_t^2 - \log S_t^2 - 1).$$

According to the results of Silvey (1970), $n(S_t^2 - \log S_t^2 - 1) \xrightarrow{\mathcal{L}} \chi^2(1)$. Also note that $n\bar{w}_t^2 \sim \chi^2(1)$ and $n(S_t^2 - \log S_t^2 - 1)$ and $n\bar{w}_t^2$ are independent. So $l_t \xrightarrow{\mathcal{L}} \chi^2(2)$ as $t \rightarrow \infty$.

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Table 1. The control limits of SSELR chart when $\lambda = 0.2$

n	IC ARL					
	100	200	300	370	400	500
1	1.8818	2.0332	2.1211	2.1558	2.1797	2.2187
2	1.4531	1.5303	1.5768	1.5996	1.6045	1.6328
3	1.3059	1.3555	1.3857	1.4033	1.4092	1.4297
4	1.2305	1.2725	1.2924	1.3071	1.3085	1.3239
5	1.1855	1.2165	1.2363	1.2456	1.2481	1.2603
6	1.1553	1.1816	1.1960	1.2053	1.2070	1.2168
7	1.1318	1.1556	1.1680	1.1761	1.1785	1.1865
8	1.1162	1.1368	1.1482	1.1553	1.1562	1.1633
9	1.1035	1.1210	1.1328	1.1372	1.1397	1.1450
10	1.0930	1.1091	1.1172	1.1232	1.1250	1.1308
11	1.0850	1.0999	1.1078	1.1125	1.1135	1.1189
12	1.0777	1.0916	1.0992	1.1030	1.1047	1.1095
13	1.0717	1.0842	1.0919	1.0952	1.0964	1.1007
14	1.0664	1.0783	1.0848	1.0879	1.0900	1.0940
15	1.0625	1.0732	1.0793	1.0823	1.0834	1.0877

Table 2. OC ARLs of SSELR (with different τ) and ELR, MEW chart with true parameters

δ	γ	τ						ELR	MEW
		10	25	50	100	250	500		
0	0.2	4.864	4.805	4.876	4.825	4.835	4.825	3.82	2.48
	0.6	29.753	9.409	9.088	8.446	8.508	8.363	7.36	8.99
	1.0	368.063	368.339	378.359	375.032	370.285	374.596	370.2	370.1
	1.4	179.177	51.912	17.803	11.419	10.423	10.029	8.48	9.36
	1.8	16.581	4.756	4.584	4.196	4.299	4.172	3.17	3.85
0.50	0.2	4.733	4.760	4.668	4.729	4.744	4.675	3.71	2.47
	0.6	8.276	6.817	6.573	6.397	6.330	6.312	5.35	7.12
	1.0	128.614	44.336	16.425	11.926	10.965	10.928	9.43	9.21
	1.4	69.485	10.530	7.108	6.590	6.226	6.333	5.14	5.98
	1.8	8.096	4.346	3.978	3.783	3.844	3.829	2.80	3.48
1.00	0.2	4.189	4.160	4.048	4.115	4.082	4.045	3.06	2.36
	0.6	4.654	4.488	4.402	4.359	4.327	4.308	3.31	3.23
	1.0	6.945	4.806	4.528	4.454	4.338	4.408	3.40	3.38
	1.4	5.996	4.187	4.044	3.945	3.899	3.819	2.83	3.28
	1.8	3.849	3.442	3.284	3.194	3.204	3.238	2.14	2.69
1.50	0.2	3.383	3.272	3.235	3.193	3.213	3.171	2.19	1.96
	0.6	3.410	3.251	3.214	3.216	3.198	3.172	2.18	2.14
	1.0	3.342	3.246	3.137	3.132	3.107	3.071	2.08	2.19
	1.4	3.223	3.008	2.961	2.940	2.945	2.912	1.88	2.21
	1.8	2.984	2.800	2.727	2.734	2.719	2.684	1.66	2.10
2.00	0.2	2.719	2.679	2.639	2.648	2.629	2.663	1.66	1.66
	0.6	2.700	2.625	2.616	2.577	2.561	2.587	1.58	1.69
	1.0	2.605	2.575	2.523	2.490	2.528	2.493	1.50	1.70
	1.4	2.615	2.490	2.473	2.458	2.455	2.420	1.42	1.72
	1.8	2.470	2.406	2.434	2.396	2.376	2.381	1.36	1.69
2.50	0.2	2.199	2.155	2.108	2.081	2.083	2.076	1.08	1.35
	0.6	2.215	2.150	2.156	2.126	2.142	2.124	1.13	1.38
	1.0	2.260	2.175	2.171	2.182	2.198	2.166	1.16	1.39
	1.4	2.239	2.199	2.187	2.167	2.161	2.199	1.17	1.41
	1.8	2.265	2.209	2.197	2.188	2.201	2.176	1.16	1.42
3.00	0.2	2.028	1.998	1.998	1.997	1.996	1.998	1.00	1.12
	0.6	2.019	2.009	2.004	1.998	2.003	2.006	1.00	1.14
	1.0	2.051	2.042	2.025	2.030	2.024	2.022	1.02	1.18
	1.4	2.093	2.060	2.052	2.047	2.047	2.047	1.05	1.21
	1.8	2.091	2.079	2.065	2.070	2.048	2.060	1.06	1.23

Figure caption list:

- Figure 1. The SSELR statistics for Laboratory 1 and 2.

Table 3. Optimal parameters (λ^*, h^*, ARL^*) of SSELR when IC ARL=370.4 and $\tau = 10$

δ	γ	n=3			n=7		
		λ^*	h^*	ARL^*	λ^*	h^*	ARL^*
0	0.2	0.62	2.7357	4.489	1.00	2.9859	2.188
	0.6	0.04	1.0566	37.610	0.06	1.0410	9.604
	1.0	ALL	ALL	370.4	ALL	ALL	370.4
	1.4	0.02	1.0235	175.840	0.06	1.0410	10.705
	1.8	0.04	1.0566	32.326	0.42	1.4424	4.438
0.50	0.2	0.66	2.9082	4.144	1.00	2.9859	2.066
	0.6	0.08	1.1338	14.746	0.44	1.4687	5.170
	1.0	0.02	1.0235	124.249	0.06	1.0410	11.622
	1.4	0.02	1.0235	85.572	0.06	1.0410	6.746
	1.8	0.04	1.0566	20.175	0.42	1.4424	3.518
1.00	0.2	0.76	3.4297	3.454	0.98	2.8320	1.997
	0.6	0.30	1.6601	6.074	0.66	1.8359	3.035
	1.0	0.06	1.0954	13.230	0.42	1.4424	3.779
	1.4	0.04	1.0566	12.862	0.42	1.4424	3.402
	1.8	0.06	1.0954	8.311	0.52	1.5906	2.762
1.50	0.2	0.92	4.6601	2.713	0.90	2.4555	1.996
	0.6	0.52	2.3455	3.810	0.90	2.4555	2.091
	1.0	0.24	1.5059	4.762	0.78	2.1043	2.390
	1.4	0.18	1.3565	4.554	0.64	1.7978	2.327
	1.8	0.14	1.2617	4.303	0.78	2.1043	2.305
2.00	0.2	0.96	5.2137	2.092	0.98	2.8320	1.996
	0.6	0.74	3.3081	2.689	0.96	2.7141	1.997
	1.0	0.44	2.0786	3.080	0.90	2.4555	2.035
	1.4	0.32	1.7153	2.979	0.86	2.3276	2.028
	1.8	0.34	1.7739	2.888	0.80	2.1583	2.021
2.50	0.2	0.96	5.2137	1.998	0.90	2.4555	1.995
	0.6	0.86	4.1055	2.123	0.86	2.3276	1.995
	1.0	0.68	3.0048	2.372	0.86	2.3276	1.997
	1.4	0.54	2.4222	2.312	0.90	2.4555	1.996
	1.8	0.56	2.4948	2.222	0.90	2.4555	1.995
3.00	0.2	0.84	3.9570	1.996	0.88	2.3906	1.994
	0.6	0.88	4.2812	2.008	0.84	2.2735	1.995
	1.0	0.82	3.8108	2.187	0.54	1.6222	1.995
	1.4	0.78	3.5509	2.178	0.84	2.2735	1.995
	1.8	0.70	3.1045	2.145	0.88	2.3906	1.994

Table 4. OC ARLs of SSELR and CP with different τ

$\tau+1$		10		50		250	
δ	γ	CP	SSELR	CP	SSELR	CP	SSELR
0.0	0.51	287.6	275.9	32.3	58.5	23.2	25.6
	0.64	415.9	409.4	123.2	249.0	44.6	81.3
	0.80	480.4	474.9	393.1	431.7	186.6	359.5
	1.00	496.6	495.9	498.4	505.8	491.1	502.2
	1.25	508.7	478.7	458.1	382.0	204.0	203.9
	1.56	500.0	433.8	205.4	195.8	34.2	33.2
	1.95	471.9	365.1	30.5	56.3	14.2	12.5
0.5	0.51	180.0	175.0	20.2	23.0	16.8	17.7
	0.64	321.1	304.6	39.3	84.3	25.2	28.1
	0.80	422.3	403.2	123.1	216.2	41.4	57.7
	1.00	473.7	429.1	265.1	273.0	63.7	81.1
	1.25	492.7	427.6	250.4	209.8	47.6	47.0
	1.56	477.7	410.1	105.4	107.7	23.3	19.8
	1.95	448.1	326.3	22.8	34.0	12.4	10.3
1.0	0.51	47.2	41.9	11.0	11.6	10.0	10.4
	0.64	120.5	114.3	13.9	14.6	12.0	11.7
	0.80	228.3	207.1	18.8	21.0	14.3	12.8
	1.00	355.0	294.8	25.0	34.8	16.2	13.5
	1.25	410.6	327.2	28.6	37.4	15.7	12.8
	1.56	419.0	308.5	22.1	22.0	12.8	10.3
	1.95	407.2	263.5	13.6	11.9	9.2	7.6
1.5	0.51	12.3	12.2	7.0	7.8	6.5	7.2
	0.64	21.0	21.0	8.0	8.0	7.1	7.2
	0.80	55.2	53.6	9.0	8.4	7.8	7.2
	1.00	132.7	117.2	10.1	8.7	8.2	7.2
	1.25	230.9	178.8	10.7	8.7	8.2	7.0
	1.56	298.6	186.8	10.4	8.1	7.7	6.5
	1.95	316.9	176.8	8.8	6.8	6.7	5.7
2.0	0.51	7.2	8.8	5.0	5.7	4.6	5.1
	0.64	8.8	10.2	5.4	5.7	4.9	5.1
	0.80	13.6	15.4	5.8	5.7	5.2	5.1
	1.00	26.9	29.7	6.3	5.6	5.4	5.0
	1.25	65.8	65.1	6.6	5.6	5.4	4.9
	1.56	132.2	95.0	6.5	5.4	5.3	4.8
	1.95	178.9	92.7	6.1	5.1	5.0	4.6

Table 5. The results of the diagnosis for Laboratory 1

t	x_t	w_t	$SSELR_t$	$lr(k_t, k)$	t_μ	F_σ	P_t	P_F
1	0.82	***	***	***	***	***	***	***
2	0.40	***	***	***	***	***	***	***
3	-2.02	-1.709	1.130	***	***	***	***	***
4	-0.02	0.123	1.062	2.5708	-1.5806	2.0067	.063025	.831058
5	-2.18	-1.139	1.191	5.2949	-2.3076	1.1228	.014618	.658129
6	-0.64	-0.024	1.168	4.7449	-2.1591	0.9640	.020125	.573460
7	-0.39	0.152	1.166	4.1427	-1.9807	0.8598	.029150	.496739
8	-0.51	0.051	1.230	4.1793	-1.9223	0.7219	.032796	.385606
9	1.17	1.381	1.067	1.5458	-1.2141	1.1407	.117818	.624527
10	0.49	0.680	1.161	0.9560	-0.9468	1.0337	.176229	.559340
11	-1.77	-1.179	1.042	2.4749	-1.5429	1.1593	.067465	.625919
12	-0.64	-0.176	1.117	2.8947	-1.6802	0.9964	.052453	.521699
13	-2.30	-1.504	1.169	6.3404	-2.5013	1.3955	.009501	.733557
14	-1.55	-0.768	1.296	8.8755	-3.0213	1.4662	.002794	.758715
15	-0.90	-0.202	1.349	9.8594	-3.2481	1.3703	.001598	.715973
16	0.03	0.592	1.209	8.5212	-2.9987	1.2737	.002952	.665441
17	0.50	0.968	1.118	6.5499	-2.5734	1.2555	.008062	.649940
18	0.60	1.000	1.169	4.8673	-2.1825	1.2128	.019150	.620220
19	-0.65	-0.136	1.227	5.6765	-2.3988	1.1271	.011958	.563751
20	0.19	0.623	1.349	4.9196	-2.2334	0.9931	.017172	.471266
21	-0.38	0.081	1.466	5.5050	-2.3747	0.8904	.012613	.394190
22	-0.72	-0.242	1.522	7.1451	-2.7488	0.8979	.005365	.394622
23	-0.21	0.257	1.686	7.7550	-2.8740	0.8086	.003987	.329623
24	-0.50	-0.035	1.833	9.6926	-3.2792	0.8567	.001479	.357847
25	0.95	1.406	1.500	6.7064	-2.6487	0.7634	.006777	.297180
26	1.59	1.910	1.573	3.0331	-1.7228	0.8326	.048401	.331541
27	0.68	0.943	1.741	2.2315	-1.4425	0.5514	.080546	.184692
28	-0.34	-0.043	1.561	3.9881	-1.9950	0.5208	.028312	.178080
29	0.30	0.571	1.689	***	***	***	***	***
30	2.23	2.321	1.917	***	***	***	***	***

Table 6. The results of the diagnosis for Laboratory 2

t	x_t	w_t	$SSELR_t$	$lr(k_t, k)$	t_μ	F_σ	P_t	P_F
1	0.30	***	***	***	***	***	***	***
2	0.18	***	***	***	***	***	***	***
3	-1.61	-2.100	1.230	***	***	***	***	***
4	-0.38	-0.002	1.119	2.0125	-1.3787	1.5774	.090097	.778782
5	0.18	0.513	1.028	0.8758	-0.9030	1.3211	.187582	.713651
6	-0.38	-0.123	1.051	1.0563	-0.8839	0.8716	.192598	.529288
7	0.45	0.870	1.035	0.4634	-0.4782	0.8503	.318316	.490587
8	-0.53	-0.440	1.087	1.0649	-0.6404	0.6621	.263863	.343681
9	-0.65	-0.574	1.173	1.9136	-0.8425	0.5443	.203758	.231782
10	-0.29	-0.027	1.276	2.6750	-0.8419	0.4466	.203917	.140358
11	1.50	2.313	1.135	0.1013	-0.0652	0.8862	.474255	.452212
12	-0.74	-0.738	1.021	0.3359	-0.3241	0.7988	.374277	.376871
13	-1.10	-1.110	1.013	0.7636	-0.6974	0.7825	.245991	.354401
14	0.33	0.678	1.007	0.7179	-0.4953	0.7014	.312342	.280506
15	-0.26	-0.080	1.045	1.2048	-0.5399	0.5987	.297028	.191364
16	-0.20	0.000	1.122	1.8154	-0.5620	0.5111	.289572	.122168
17	-0.47	-0.361	1.211	2.6342	-0.7030	0.4442	.244272	.077412
18	-1.22	-1.347	1.210	3.1825	-1.1836	0.4739	.123848	.092801
19	-0.47	-0.267	1.306	4.1972	-1.3244	0.4096	.098678	.057248
20	-0.68	-0.549	1.454	5.4725	-1.5901	0.3661	.062193	.038832
21	-1.07	-1.069	1.596	6.9613	-2.1198	0.3759	.022065	.044635
22	-1.37	-1.416	1.763	9.2102	-2.9803	0.4973	.003165	.114379
23	1.44	2.342	1.100	4.1922	-1.9911	0.6497	.028755	.222653
24	-0.50	-0.239	1.030	5.5144	-2.3183	0.5948	.014453	.191864
25	-0.83	-0.647	1.047	8.6340	-3.0649	0.8774	.002582	.354929
26	0.15	0.616	1.009	9.0232	-3.1810	0.8432	.001946	.323457
27	0.84	1.463	1.064	7.0366	-2.7277	0.4958	.005747	.168408
28	0.54	1.018	1.173	***	***	***	***	***
29	2.51	3.160	2.073	***	***	***	***	***